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Diversity Combining Techniques under Employment of Generalized Receiver in Wireless Communication Systems with Rayleigh Fading Channel

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The bit error rate (BER) performance of wireless communication system employing the generalized receiver (GR) under the binary phase shift keying (BPSK) modulation over Rayleigh fading channel with three common diversity combining techniques, namely, the selection combining, equal gain combining, and maximal ratio combining with the purpose to mitigate the effects of multipath fading is investigated. Simulation demonstrates a high performance gain under employment of GR in wireless communication system in comparison with modern approaches.

Keywords: Generalized Receiver (GR), Diversity Combining, Selection Combining (SC), Equal Gain Combining (EGC), Maximal Ratio Combining (MRC), And Rayleigh Fading Channel.

1. Introduction

In urban and indoor environments, the line-of-sight between the transmitter and receiver has a random character and the transmitted signal is distorted along multiple paths. As a result, the phase shifts, time delays, attenuations, and distortions are introduced and the transmitted signals can destructively interfere with each other at the aperture of the receiving antenna. Antenna diversity is an effective solution to mitigate the multipath fading (Proakis 1995). Multiple antennas offer to the receiver several observations of the same signal and each antenna experiences a different interference source. Thus, if one antenna experiences a deep fade, it is likely that other antennas may have sufficient signal. Multiple antenna system can provide a robust radio channel transmission. Diversity combining technique is applied to combine the multiple received signals of a diversity reception device into a single improved signal. The experimental results of dual and triple diversity are presented in (Wambeck and Ross 1951). The derivation of the combining weights of the optimal linear combiner for dual diversity is discussed in (Kahn 1954), and generalization of the Kahn's results for higher orders of diversity is presented in (Brennan 1955). The technique described by Kahn and Brennan is commonly known

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as maximal ratio combining (MRC). A thorough treatment of the more commonly-used diversity combining techniques is presented in (Schwartz and Bennett 1966). The performance of diversity combiner is measured by diversity gain which is the difference in signal-to-noise ratio (SNR) between the output of a diversity combiner and the signal on a single branch measured at a given probability level. Diversity gain quantifies the improvement in SNR of a received signal that is obtained using signals from different receiver branches. Equalizers or rake receivers employed in wideband radios cannot mitigate flat fading with a single antenna (Benvenuto and Tomba 1997), but when combined with antenna diversity they can improve performance in both flat fading and frequency-selective fading channels. Predetection combining using space-diversity antenna array is commonly used to combat fading in mobile radio systems. These methods have been extensively studied in (Brennan 1959; Jakes 1971). The most widely considered predetection combining techniques are selection combining (SC), equal-gain combining (EGC), and maximal-ratio combining (MRC). SC is a suboptimal combining scheme in which the branch signal with the largest amplitude or SNR is selected for demodulation (Altman and Sichak 1956; Mack 1955; Kavehrad and McLane 1985). SC scheme is used to decrease the receiver complexity in terms of the number of radio frequency (RF) chains, proposed in (Chen and Yuan 2005; Jakes 1994). The SC scheme has been extended to the cases where the signals on more than one receive antennas with the largest instantaneous SNRs are combined (Eng, Kong, and Milstein 1996; Kong and Milstein 1999; Win, Mallik, and Chrisikos 1999; Zvonar, and Proakis 2004). This scheme is referred to as hybrid selection/maximal-ratio combining (HS/MRC) (Win, Mallik, and Chrisikos 1999). MRC is one of the most widely used diversity combining schemes in which SNR is the sum of the SNR’s of each individual diversity branch. It is the optimal but the most complex combining model, since MRC requires cognition of all channels fading parameters (Suzuki 1977).The bit error rate (BER) of MRC receiver for BPSK signals in the presence of log-normal and Rayleigh fading is derived in (Nikolic, Milic, Krstic, and Spalevic 2011). The BER of MRC receiver is presented in the presence of log-normal and Rice fading in (Nikolic, Krstic, Milic, and Arsic 2008); and Nakagami-*m* fading and shadowing in (Nikolic, Krstic, Stefanovic, Panic, and Destovic 2010). The performance analysis of MRC receiver in the presence of Weibull fading and shadowing are described by both log-normal and Gamma distributions in (Nikolic, Krstic, Popovic, Stefanovic, and Stefanovic 2010). The performance analysis of EGC has been extensively reported for the case when channel information is perfectly known at the receiver (Beaulieu and Abu-Dayya 1991; Zhang 1999; Annamalai, Tellambura, and Bhargava 2000; Alouini and Simon 2001; Qi, Alouini, and Ko 2003). To evaluate the performance of the EGC, a decision variable based approach is presented in (Annamalai, Tellambura, and Bhargava 2000; Alouini and Simon 2001) and the simplifying analysis of that performance is presented in (Beaulieu and Abu-Dayya 1991). Closed form error probability expressions for the case of dual branch EGC over Rayleigh fading channels have been presented in (Qi, Alouini, and Ko 2003).

In this paper we investigate an attempt to implement the GR, which is constructed based on the generalized approach to signal processing (GASP) in noise (Tuzlukov 1998, 2001) in wireless communication system with one transmit antenna and *N* receive antennas. The generalized receiver (GR) is a combination of the Neyman-Pearson (NP) receiver and energy receiver (ER) based on the generalized approach to signal processing in noise (Tuzlukov 1998, 2001). The NP receiver is optimal for detection of signals with known parameters, and the ER is optimal for detection of signals with unknown parameters. The idea to employ GR in MIMO system is mentioned in (Tuzlukov 2010, 2012). The attempt to analyze an employment of GR with ZF and MMSE equalizers in MIMO wireless communication systems is discussed in (Das, Shbat, and Tuzlukov 2012). Based on signal subspace estimation, GR can be estimated from the received signal with a prior knowledge of signature waveform (Tuzlukov 2010). Multiuser GR is used for uniformly quantized CDMA signals based on the generalized approach to signal processing in noise (Tuzlukov 2008). We study the SC, MRC and EGC techniques used by GR under BPSK modulation over Rayleigh fading channel. The BER performance of wireless communication system over Rayleigh fading channel employing the GR with discussed diversity combining techniques has a great advantage in comparison with widely used receivers, for example, receiver based on the Neyman Pearson criterion employing the same diversity combining procedures.

The paper is organized as follows. In section 2 we briefly describe the generalized receiver (GR). Diversity combining techniques for wireless communication system are discussed in section 3. Simulation results are given in section 4 and some conclusions are made in section 5.

2. Generalized Receiver

The GR can be simply presented in a form of block diagram shown in Fig.1, (Tuzlukov 2011). In this flowchart, MSG is the model signal generator (local oscillator), PF is the preliminary filter with the impulse response $h_{PF}(t)$, and AF is the additional filter with the impulse response $h_{AF}(t)$. A resonant frequency of the AF is detuned relative to a resonant frequency of PF on such a value that, at the PF output both the signal and noise can be appeared whereas *the only noise* is appeared at the AF output. A value of detuning between the AF and PF resonant frequencies is more than $4-5\Delta f_a$, where Δf_a is the signal bandwidth. In this case the coefficient of correlation is not more than 0.05. If the Gaussian noise $w(t)$ comes in at the AF and PF inputs (the GR linear system front end), the noise forming at the AF and PF outputs is Gaussian, too, because the AF and PF are the linear systems and, in a general case, takes the following form (Tuzlukov 2001, 2011):

$$\xi_{PF}(t) = \int_{i=-\infty}^{\infty} h_{PF}(\tau) n_i(t-\tau)\, d\tau \,, \tag{1}$$

$$\xi_{AF}(t) = \int_{i=-\infty}^{\infty} h_{PF}(\tau) n_i(t-\tau)\, d\tau \,. \tag{2}$$

If the additive white Gaussian noise (AWGN) with zero mean and two-sided power spectral density $N_0/2$ is coming in at the AF and PF inputs (the GR linear system front end), then the noise forming at the AF and PF outputs is Gaussian with zero mean and variance given by,

$$\sigma_n^2 = \frac{N_0\omega_0}{8\Delta_F} \tag{3}$$

where Δ_F is the AF or (PF) bandwidth and ω_0 is the resonance frequency. The main functioning condition of GR is the equality over the whole range of parameters between the model signal x_k^m forming at the GR MSG output for user k and the expected signal x_k forming at the GR input linear system (the PF) output i.e.,

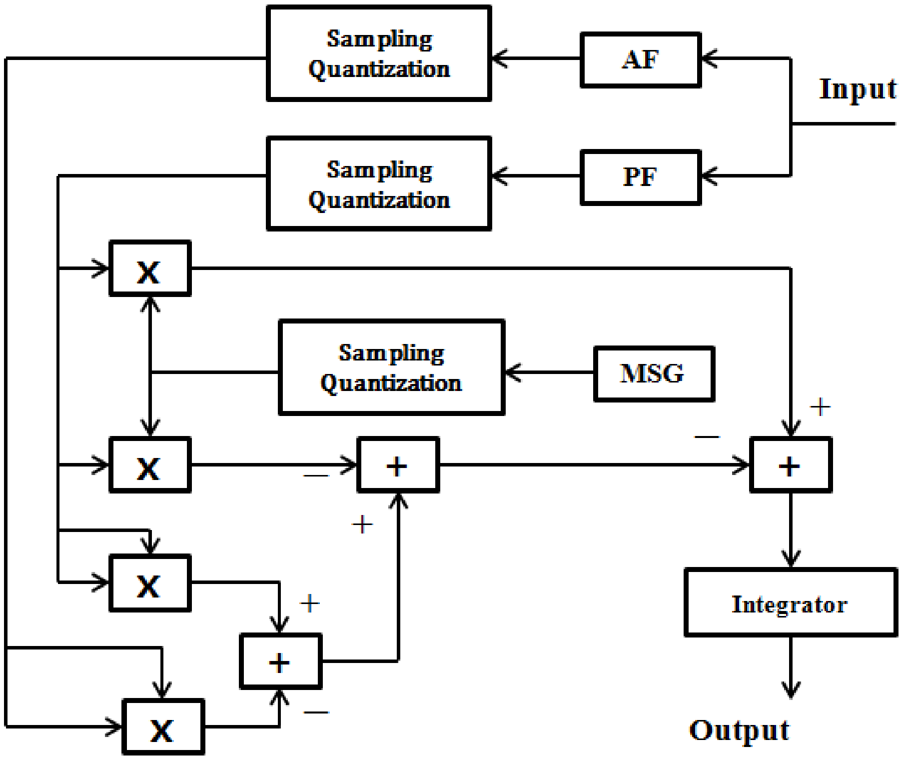

Figure 1: Principal GR Flowchart

$$x_k(t) = x_k^m(t) \tag{4}$$

The stochastic process at the output of the PF takes the following form

$$X_{PF}(t) = x_i(t) + \xi_{PF}(t),\ i \in [1, K];\ 0 \le t \le T \tag{5}$$

where *K* is the sample size; [0,*T*] is the time interval within the limits of which the input stochastic process is observed; $\xi_{PF}(t)$ is the Gaussian noise at the output of the PF. The process at the GR output takes the following form:

$$Z_g^{out} = 2\sum_{i=1}^{N} x_i^m(t)x_i(t) - \sum_{i=1}^{N} x_i^2(t) + 2\sum_{i=1}^{N} x_i^m(t)\xi_{PF}(t) - 2\sum_{i=1}^{N} x_i(t)\xi_{PF}(t)$$

$$-\sum_{i=1}^{N}\xi_{PF}^2(t) + \sum_{i=1}^{N}\xi_{AF}^2(t), \tag{6}$$

where $\xi_{AF}(t)$ is the noise at the output of the AF. Satisfying the main GR functioning condition given by (4) we obtain,

$$Z_g^{out} = \sum_{i=1}^{N} x_i^2(t) + \sum_{i=1}^{N} \xi_{AF}^2(t) - \sum_{i=1}^{N} \xi_{PF}^2(t) . \tag{7}$$

The total background noise which is formed at the output of the multiuser generalized receiver takes the following form:

$$n_b = \xi_{AF}^2 - \xi_{PF}^2 . \tag{8}$$

3. Diversity Combining

In this paper, we consider one transmit antenna and *N* receive antennas over flat fading channel shown in Fig. 2. For the $i\text{th}$ receive antenna, each transmitted signal is multiplied by a random channel gain h_i. As the channel under consideration is a Rayleigh channel, the real and imaginary parts of h_i are Gaussian distributed with the mean zero and variance 0.5.

Consider a single-user system model wherein the received signal is a sum of the desired signal and noise:

$$\mathbf{y} = \mathbf{H}x + \mathbf{n} , \tag{9}$$

where x is the transmitted signal, $\mathbf{H}$ is the channel gain and $\mathbf{n}$ is the white Gaussian noise. The power of the signal over a single symbol period T, at $m\text{th}$ element takes the following form

$$P = \frac{1}{T}\int_0^T \left|h_m(t)\right|^2 \left|x(t)\right|^2 dt = \left|h_m(t)\right|^2 \frac{1}{T}\int_0^T \left|x(t)\right|^2 dt = \left|h_m\right|^2 , \tag{10}$$

where the term $\left|h_m(t)\right|$ remains constant over a symbol period and can be brought out of the integral and $x(t)$ is assumed to have unit power. We assume that, $E\left\{\left|n_m(t)\right|^2\right\} = \sigma_g^2$ and we get the instantaneous bit energy to noise ratio for NP receiver at the $i\text{th}$ receive antenna as follows (Brennan 1955; Jakes 1971):

$$\gamma_i^{NP} = \frac{\left|h_i\right|^2}{\sigma_g^2} , \tag{11}$$

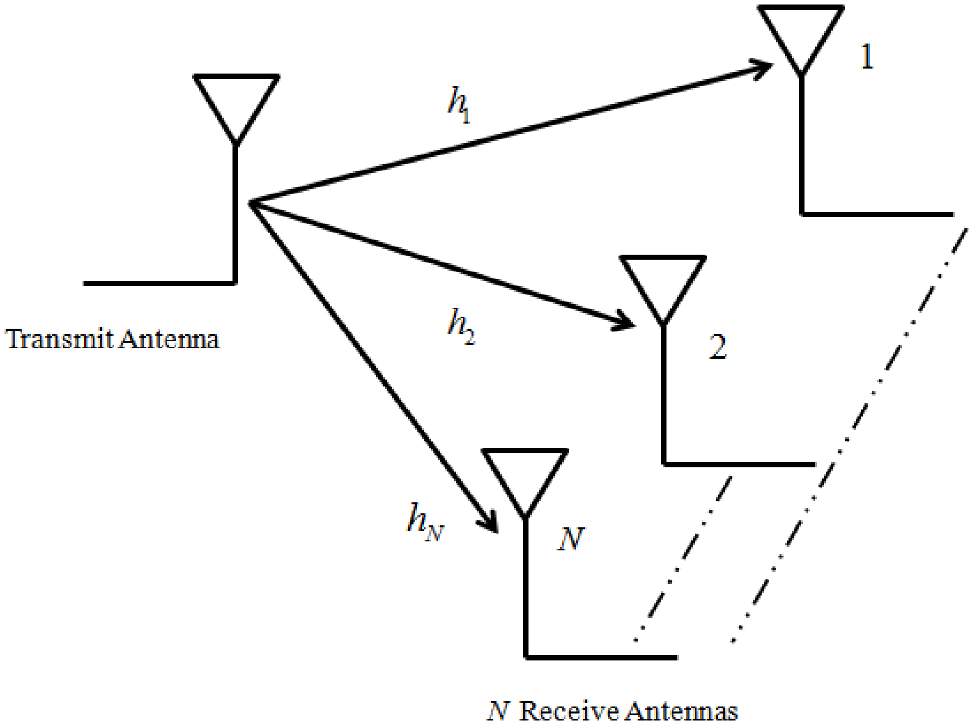

Figure 2: Receive Diversity in Wireless Communication

where σ_g^2 is the variance of the white Gaussian noise. For GR the variance of the background noise is $\sigma_g^2 = 4\sigma_n^2$ (Tuzlukov 2001, 2011). The bit energy to ratio at the ith receive antenna takes the following form

$$\gamma_i^{GR} = \frac{|h_i|^2}{4\sigma_n^4}\,. \tag{12}$$

In the case of NP receiver the probability density function (pdf) of γ_i takes the following form (Brennan 1955; Jakes 1971):

$$p(\gamma_i^{NP}) = \frac{1}{\sigma_n^2} e^{\frac{-\gamma_i^{NP}}{\sigma_n^2}}\,. \tag{13}$$

The pdf of γ_i for GR (Tuzlukov 2001, 2011) is defined as follows:

$$P(\gamma_i^{GR}) = \frac{1}{4\sigma_n^4} e^{\frac{-\gamma_i^{GR}}{4\sigma_n^4}}\,. \tag{14}$$

3.1. Selection Combining

In selection combining technique, the receiver selects the antenna with the highest received signal power and ignores observations from the other antennas. As each element is an independent sample of the fading process, the element with the greatest SNR is chosen for further processing.

Outage probability is the probability that the bit energy to noise ratio falls below a threshold. The outage probabilities on the $i\text{th}$ receive antenna for NP receiver takes the following form (Kavehard 1985; Chen 2005):

$$P_{out}^{NP} = P[\gamma_i^{NP} < \gamma_s] = \int_0^s \frac{1}{\sigma_n^2} e^{\frac{-\gamma_i^{NP}}{\sigma_n^2}} d\gamma_i^{NP} = 1 - e^{\frac{-\gamma_s}{\sigma_n^2}} \tag{15}$$

where, γ_s is the defined threshold for bit energy to noise ratio.

The outage probabilities by GR on the $i\text{th}$ receive antenna can be defined as follows (Tuzlukov 2001, 2011):

$$P_{out}^{GR} = P[\gamma_i^{GR} < \gamma_s] = \int_0^s \frac{1}{4\sigma_n^4} e^{\frac{-\gamma_i^{GR}}{4\sigma_n^4}} d\gamma_i^{GR} = 1 - e^{\frac{-\gamma_s}{4\sigma_n^4}} . \tag{16}$$

In N antenna case, the probability that all bit energy to noise ratio on all the receive antennas are below the threshold γ_s, i.e.

$$P_{out} = P[\gamma_1, \gamma_2, \gamma_3, ..., \gamma_N < \gamma_s], \tag{17}$$

where, $\gamma_1, \gamma_2, \gamma_3, ..., \gamma_N$ are the bit energy to noise ratio on the 1st, 2nd, 3rd, and so on till the Nth receive antenna. Since the channel on each antenna is assumed to be independent, the joint probability is the product of individual probabilities.

$$P[\gamma_2 < \gamma_s] P[\gamma_3 < \gamma_s] ..... P[\gamma_N < \gamma_s] = \prod_{i=1}^{N} P[\gamma_i < \gamma_s] . \tag{18}$$

For NP receiver, the joint probability is as follows (Kavehard 1985; Chen 2005):

$$P_{joint}^{NP} = \left[1 - e^{\frac{-\gamma_i^{NP}}{\sigma_n^2}} \right]^N . \tag{19}$$

For GR it takes the following form (Tuzlukov 2001, 2011):

$$P_{joint}^{GR} = \left[1 - e^{\frac{-\gamma_i^{GR}}{4\sigma_n^4}} \right]^N . \tag{20}$$

P_{out} also represents the cdf of the output SNR as a function of the threshold γ_s.

Therefore the pdf of the output SNR is as follows:

$$p(\gamma)=\frac{dP_{out}}{d\gamma}. \tag{21}$$

The pdf of the output SNR for NP receiver takes the following form (Kavehard 1985; Chen 2005):

$$p(\gamma^{NP})=\frac{N}{\sigma_n^2}e^{\frac{-\gamma^{NP}}{\sigma_n^2}}\left[1-e^{\frac{-\gamma^{NP}}{\sigma_n^2}}\right]^{N-1}. \tag{22}$$

For GR, the pdf of the output SNR is as follows (Tuzlukov 2001, 2011):

$$p(\gamma^{GR})=\frac{N}{4\sigma_n^4}e^{\frac{-\gamma^{GR}}{4\sigma_n^4}}\left[1-e^{\frac{-\gamma^{GR}}{4\sigma_n^4}}\right]^{N-1}. \tag{23}$$

The average output bit energy to noise ratio can be defined as follows:

$$E(\gamma)=\int_0^\infty \gamma p(\gamma)d\gamma. \tag{24}$$

In case of NP receiver, the average bit energy to noise ratio is as follows (Kavehard 1985; Chen 2005):

$$E(\gamma^{NP})=\int_0^\infty \gamma^{NP}\frac{N}{\sigma_n^2}e^{\frac{-\gamma^{NP}}{\sigma_n^2}}\left[1-e^{\frac{-\gamma^{NP}}{\sigma_n^2}}\right]^{N-1}d\gamma^{NP}=\sigma_n^2\sum_{i=1}^{N}\frac{1}{i}. \tag{25}$$

For GR, the average bit energy to noise ratio takes the following form (Tuzlukov 2001, 2011):

$$E(\gamma^{GR})=\int_0^\infty \gamma^{GR}\frac{N}{4\sigma_n^4}e^{\frac{-\gamma^{GR}}{4\sigma_n^4}}\left[1-e^{\frac{-\gamma^{GR}}{4\sigma_n^4}}\right]^{N-1}d\gamma^{GR}=2\sigma_n^4\sum_{i=1}^{N}\frac{1}{i}. \tag{26}$$

The effective bit energy to noise ratio with selection diversity is the integral of the conditional BER integrated over all possible values of γ. The total BER for NP receiver takes the following form (Kavehard 1985; Chen 2005):

$$\begin{aligned}P_e^{NP}&=\int_0^\infty \frac{1}{2}erfc\left(\sqrt{\gamma^{NP}}\right)p\left(\gamma^{NP}\right)d\gamma^{NP}\\&=\int_0^\infty \frac{1}{2}erfc\left(\sqrt{\gamma^{NP}}\right)\frac{N}{\sigma_n^2}e^{\frac{-\gamma^{NP}}{\sigma_n^2}}\left[1-e^{\frac{-\gamma^{NP}}{\sigma_n^2}}\right]^{N-1}d\gamma^{NP}.\end{aligned} \tag{27}$$

The total BER for GR takes the following form (Tuzlukov 2001, 2011):

$$P_e^{GR} = \int_0^\infty \frac{1}{2} erfc\left(\sqrt{\gamma^{GR}}\right) p\left(\gamma^{GR}\right) d\gamma^{GR}$$

$$= \int_0^\infty \frac{1}{2} erfc\left(\sqrt{\gamma^{GR}}\right) \frac{N}{4\sigma_n^4} e^{\frac{-\gamma^{GR}}{4\sigma_n^4}} \left[1 - e^{\frac{-\gamma^{GR}}{4\sigma_n^4}}\right]^{N-1} d\gamma^{GR}. \quad (28)$$

Finally the bit error probability with selection diversity for NP receiver takes the following form (Kavehard 1985; Chen 2005):

$$P_e^{NP} = \frac{1}{2} \sum_{k=0}^{N} (-1)^k \binom{N}{k} \left(1 + \frac{k}{\sigma_n^2}\right)^{-1/2}. \quad (29)$$

For GR, the bit error probability with selection diversity takes the following form (Tuzlukov 2001, 2011):

$$P_e^{GR} = \frac{1}{2} \sum_{k=0}^{N} (-1)^k \binom{N}{k} \left(1 + \frac{k}{4\sigma_n^4}\right)^{-1/2}. \quad (30)$$

3.2. Maximal Ratio Combining

On the $i\text{th}$ receive antenna, the received signal is defined as,

$$y_i = h_i x + n_i, \quad (31)$$

where y_i is the received signal, h_i is the channel gain, x is the transmitted signal and n_i is the noise on $i\text{th}$ receive antenna. The received signal can be presented in the matrix form as follows:

$$\mathbf{y} = \mathbf{h}x + \mathbf{n}, \quad (32)$$

where $\mathbf{y} = \left[y_1 y_2 .....y_N\right]^T$ is the received signal from all the receive antennas, $\mathbf{h} = \left[h_1 h_2 ......h_N\right]^T$ is the channel gain on all the receive antennas, x is the transmitted signal, and $\mathbf{n} = \left[n_1 n_2 ....n_N\right]^T$ is the noise on all the receive antennas. We can define the equalized signal as follows:

$$\hat{x} = \frac{\mathbf{h}^H \mathbf{y}}{\mathbf{h}^H \mathbf{h}} = \frac{\mathbf{h}^H \mathbf{y} x}{\mathbf{h}^H \mathbf{h}} + \frac{\mathbf{h}^H \mathbf{n}}{\mathbf{h}^H \mathbf{h}} = x + \frac{\mathbf{h}^H \mathbf{n}}{\mathbf{h}^H \mathbf{h}}, \quad (33)$$

where $\mathbf{h}^H \mathbf{h} = \sum_{i=1}^{N} \left|h_i\right|^2$ is the sum of the channel powers across all the receive antennas. The effective bit energy to noise ratio for NP receiver takes the following form (Nikolic 2011):

$$\gamma^{NP} = \sum_{i=1}^{N} \frac{\left|h_i\right|^2}{\sigma_n^2} = N\gamma_i^{NP} . \tag{34}$$

The effective bit energy to noise ratio for GR is defined as follows (Tuzlukov 2001, 2011):

$$\gamma^{GR} = \frac{1}{2}\sum_{i=1}^{N} \frac{\left|h_i\right|^2}{4\sigma_n^4} = N\gamma_i^{GR} . \tag{35}$$

The pdf of γ^{NP} for the NP receiver is as follows (Nikolic 2011):

$$P\left(\gamma^{NP}\right) = P\left(\gamma < \gamma_s\right) = \int_0^{\gamma_s} \frac{1}{(N-1)!\left(\sigma_n^2\right)^N}\left(\gamma^{NP}\right)^{N-1} e^{\frac{-\gamma^{NP}}{\sigma_n^2}} , \quad \gamma^{NP} \geq 0 . \tag{36}$$

For GR, the pdf of γ^{GR} takes the following for(Tuzlukov 2001, 2011):

$$p\left(\gamma^{GR}\right) == \int_0^{\gamma_s} \frac{1}{(N-1)!\left(4\sigma_n^4\right)^N}\left(\gamma^{GR}\right)^{N-1} e^{\frac{-\gamma^{NP}}{4\sigma_n^4}} , \quad \gamma^{GR} \geq 0 \tag{37}$$

The total bit error rate for NP receiver takes the following form (Nikolic 2011):

$$\begin{aligned} P_e^{NP} &= \int_0^{\infty} \frac{1}{2} erfc\left(\sqrt{\gamma^{NP}}\right) p\left(\gamma^{NP}\right) d\gamma^{NP} \\ &= \int_0^{\infty} \frac{1}{2} erfc\left(\sqrt{\gamma^{NP}}\right) \frac{1}{(N-1)!\left(\sigma_n^2\right)^N}\left(\gamma^{NP}\right)^{N-1} e^{\frac{-\gamma^{NP}}{\sigma_n^2}} d\gamma^{NP} . \end{aligned} \tag{38}$$

The total bit error rate for GR takes the following form (Tuzlukov 2001, 2011):

$$\begin{aligned} P_e^{GR} &= \int_0^{\infty} \frac{1}{2} erfc\left(\sqrt{\gamma^{GR}}\right) p\left(\gamma^{GR}\right) d\gamma^{GR} \\ &= \int_0^{\infty} \frac{1}{2} erfc\left(\sqrt{\gamma^{GR}}\right) \frac{1}{(N-1)!\left(4\sigma_n^4\right)^N}\left(\gamma^{GR}\right)^{N-1} e^{\frac{-\gamma^{GR}}{4\sigma_n^4}} d\gamma^{GR} . \end{aligned} \tag{39}$$

Finally we can write the total bit error rate in a simplified form as follows:

$$P_e = p^N \sum_{k=0}^{N-1} \binom{N-1+k}{k} (1-p)^k , \tag{40}$$

where $p = \frac{1}{2} - \frac{1}{2}\left(1 + \frac{1}{\sigma_n^2}\right)^{-1/2}$, for the NP receiver and in case of the GR

$$p=\frac{1}{2}-\frac{1}{2}\left(1+\frac{1}{4\sigma_n^4}\right)^{-1/2}.$$

3.3. Equal Gain Combining

The effective signal to noise ratio with EGC is the channel power accumulated over all receive chains. The EG combiner takes the following form

$$\mathbf{w}^H\mathbf{h}=\sum_{i=1}^{N}|h_i|. \tag{41}$$

The noise and instantaneous *SNR* are given by

$$n=\mathbf{w}^H\mathbf{w}\sigma_n^2=N\sigma_n^2, \tag{42}$$

where $(\ldots)^H$ is the Hermitian transpose operation.

$$\gamma=\frac{\left[\sum_{i=1}^{N}|h_i|\right]^2}{N\sigma_n^2}. \tag{43}$$

For NP receiver the effective SNR with EGC takes the following form (Beaulieu 1991; Zhang 1999):

$$\begin{aligned}E\left(\gamma_i^{NP}\right)&=\frac{E\left\{\left(\sum_{i=1}^{N}|h_i|^2\right)\right\}}{2N\sigma_n^2}=\frac{1}{2N\sigma_n^2}E\left(\sum_{i=1}^{N}\sum_{k=1}^{N}|h_i||h_k|\right)\\&=\frac{1}{2N\sigma_n^2}\left[E\left\{\sum_{i=1}^{N}|h_i|^2\right\}+E\left\{\sum_{i=1}^{N}\sum_{k=1,k\neq i}^{N}|h_i||h_k|\right\}\right]\\&=\frac{1}{2\sigma_n^2}\left[\sum_{i=1}^{N}E\left\{|h_i|^2\right\}+\sum_{i=1}^{N}\sum_{k=1,k\neq i}^{N}E\left\{|h_i|\right\}E\left\{|h_k|\right\}\right].\end{aligned} \tag{44}$$

The effective SNR with EGC for GR takes the following form (Tuzlukov 2001, 2011):

$$\begin{aligned}E\left(\gamma_i^{GR}\right)&=\frac{E\left\{\left(\sum_{i=1}^{N}|h_i|^2\right)\right\}}{8N\sigma_n^4}=\frac{1}{8N\sigma_n^4}E\left(\sum_{i=1}^{N}\sum_{k=1}^{N}|h_i||h_k|\right)\\&=\frac{1}{8N\sigma_n^4}\left[E\left\{\sum_{i=1}^{N}|h_i|^2\right\}+E\left\{\sum_{i=1}^{N}\sum_{k=1,k\neq i}^{N}|h_i||h_k|\right\}\right]\end{aligned}$$

$$=\frac{1}{8\sigma_n^4}\left[\sum_{i=1}^{N}E\left\{|h_i|^2\right\}+\sum_{i=1}^{N}\sum_{k=1,k\neq i}^{N}E\left\{|h_i|\right\}E\left\{|h_k|\right\}\right], \tag{45}$$

where, the first term is a chi-square random variable with $2N$ degrees of freedom having mean value of $2N\sigma_{h_i}^2$. Hence the first term reduces to the following form:

$$\sum_{i=1}^{N}|h_i|^2=N\,. \tag{46}$$

The second term is a product of two Rayleigh random variables. The mean of Rayleigh random variable with variance $\sigma_{h_h}^2$ is $\sigma_{h_i}\times\sqrt{\frac{\pi}{2}}$. Hence the second term takes the following form:

$$\sum_{i=1}^{N}\sum_{k=1,k\neq n}^{N}|h_i||h_k|=N\sqrt{\frac{\pi}{4}}\left(N-1\right)\sqrt{\frac{\pi}{4}}=N\left(N-1\right)\frac{\pi}{4}\,. \tag{47}$$

Simplifying the effective signal to noise ratio with EGC for NP receiver takes the following form (Zhang 1999; Annamalai 2000):

$$E\left(\gamma_i^{NP}\right)=\frac{1}{N\sigma_n^2}\left[N+N\left(N-1\right)\frac{\pi}{4}\right]$$

$$=\frac{1}{\sigma_n^2}\left[1+\left(N-1\right)\frac{\pi}{4}\right]. \tag{45}$$

The effective signal to noise ratio with equal gain combining for GR (Tuzlukov 2001, 2011) is simplified in the following form:

$$E\left(\gamma_i^{GR}\right)=\frac{1}{4N\sigma_n^4}\left[N+N\left(N-1\right)\frac{\pi}{4}\right]=\frac{1}{4N\sigma_n^4}\left[1+\left(N-1\right)\frac{\pi}{4}\right]. \tag{48}$$

There is no closed form solution for the BER for general *N*, but several researchers have investigated the BER performance in several kinds of fading channels (Zhang 1999; Annamalai and Tellambura 2000). The closed form solutions in Rayleigh fading for *N*=2 and *N*=3 based on the characteristic function method is presented in (Zhang 1999).

4. Simulation

The simulation is carried out using MATLAB. The BER performance comparison between NP receiver and GR as a function of signal to noise ratio under BPSK modulation with diversity combining techniques in wireless communication system is presented. We observe that performance improvement can be achieved in wireless communication system employing GR in comparison with NP receiver for different diversity combining techniques. For selection combining with single receive antenna, at $SNR=10$dB, the employment of GR in wireless communication system gives us

$BER_{GR,SC}^{R_1} = 9.315\times10^{-3}$ whereas, NP has $BER_{NP,SC}^{R_1} = 23.07\times10^{-3}$, Fig. 3. In the case when the number of receive antenna is 2, NP has $BER_{NP,SC}^{R_2} = 2.966\times10^{-3}$ and GR has $BER_{GR,SC}^{R_2} = 0.979\times10^{-3}$ for SC technique, Fig. 3. For MRC with single receive antenna, $BER_{GR,MRC}^{R_1} = 9.437\times10^{-3}$ is achieved using GR and $BER_{NP,MRC}^{R_1} = 23.39\times10^{-3}$ for the NP, Fig. 4. Using antenna diversity, as an example, using 2 receive antennas with MRC, $BER_{GR,MRC}^{R_2} = 0.415\times10^{-3}$ is achieved by GR and $BER_{NP,MRC}^{R_2} = 1.578\times10^{-3}$ for the NP receiver, Fig. 4. Moreover, using single receive antenna with EGC, we achieve $BER_{GR,EGC}^{R_1} = 9.451\times10^{-3}$ by GR and $BER_{NP,EGC}^{R_1} = 23.09\times10^{-3}$ for NP receiver, Fig. 5. On the other hand, using antenna diversity with EGC, $BER_{GR,EGC}^{R_2} = 2.158\times10^{-3}$ is achieved using GR and $BER_{NP,EGC}^{R_2} = 0.545\times10^{-3}$ for the NP receiver, Fig. 5. In Fig. 6, the BER performance comparison between GR and NP receiver as a function of signal to noise ratio under BPSK modulation with SC, MRC, and EGC techniques are presented using 2 receive antennas.

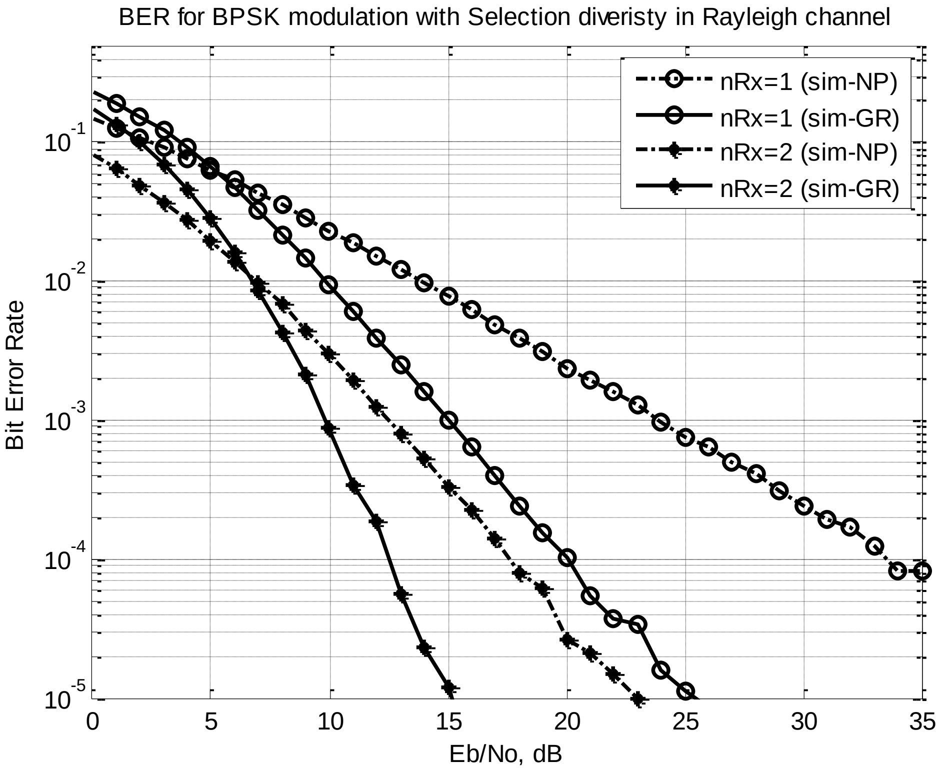

Figure 3: BER for BPSK Modulation with Selection Combining In Rayleigh Channel

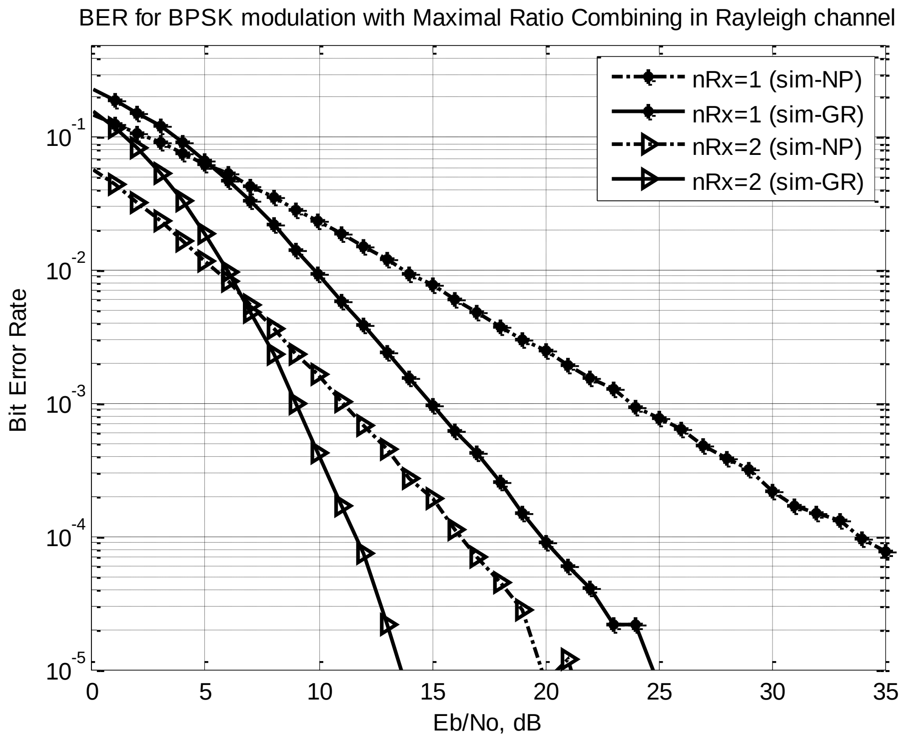

Figure 4: BER for BPSK Modulation with Maximal Ratio Combining In Rayleigh Channel

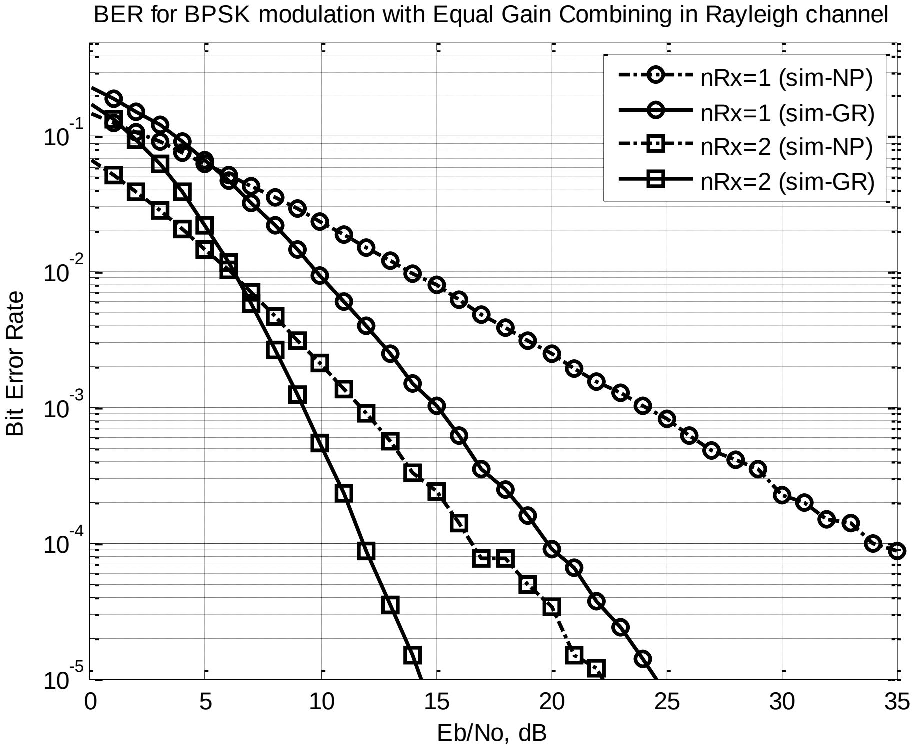

Figure 5: BER for BPSK Modulation with Equal Gain Combining In Rayleigh Channel

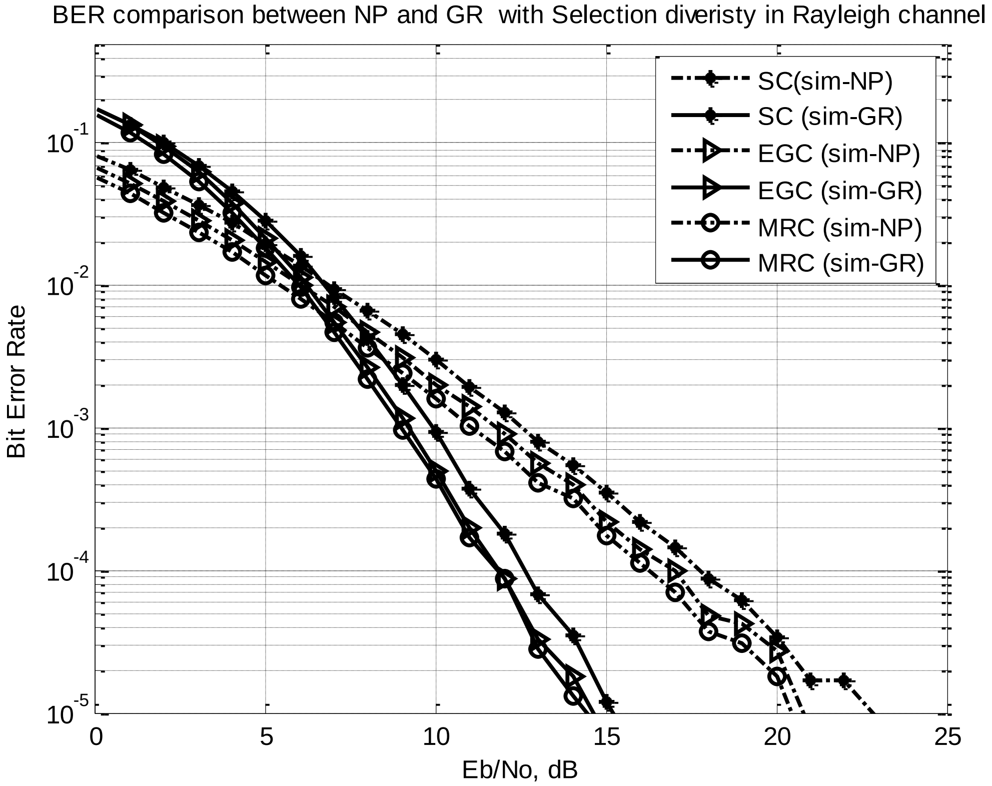

Figure 6: BER Performance Comparison between NP and GR for BPSK Modulation with SC, EGC and MRC over Rayleigh Channel.

5. Conclusion

In this paper, we investigate an attempt to employ the GR with three diversity combining techniques in wireless communication system. BER performances under BPSK modulation over Rayleigh fading channel in wireless communication system with diversity combining techniques are discussed. The performance comparison between GR and NP receiver with diversity combining techniques such as SC, EGC, and MRC techniques demonstrates the low BER by the employment of GR in wireless communication system. We can observe the lesser losses in SNR caused by implementation of diversity combining techniques in wireless communication system employing GR in comparison with the use of NP receiver.

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