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# Multi-Objective Linear Programming for Project Time-Cost Optimization by Fuzzy Goal Programming with Genetic Algorithm

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*This paper work demonstrated a goal programming approach for solving multi-objective project management problem where the goals of the decision maker are fuzzy by genetic algorithm. The project execution time is a major concern to the involved stakeholders (client, contractors and consultants). For optimization of total project cost through time control, crashing cost is considered here as a critical factor. The proposed approach aims to formulate a multi objective linear programming model to simultaneously minimize the total project cost, completion time and crashing cost within the framework of the satisfaction level of decision maker with fuzzy goal and fuzzy cost coefficients. To make such problems realistic, triangular fuzzy numbers and the concept of minimum accepted level method are employed to formulate the problem. The proposed model leads decision makers to choose the desired compromise solution under different risk levels and the project optimization problems have been solved under multiple uncertainty conditions. Minimum operator and weighted average operator method is used to aggregate all fuzzy set of the problem with linear membership functions in order to develop a better representation of fuzzy project management model and the solution is obtained by using MATLAB.*

**Key Words:** Project Management, Multi-Objective Linear Programming, Weighted Average Operator, Fuzzy Goal Programming and Fuzzy Based *Genetic Algorithm*

## 1. Introduction

Project management is an activity to ensure smooth implementation of any as per its specification. A project is a combination of interrelated activities which must be executed in a certain order which is known as precedence relationship before the entire task is completed. It is the process of planning; scheduling and controlling projects. Planning phase involves clearly defined goals and objectives of the project; scheduling phase involves determining the time and sequence interdependencies between project activities; and the control phase involves dealing with unexpected events in order to maintain the time and budget requirements. Therefore, it is truly important for project managers to confirm the project completion that includes quality, effectiveness, the specified completion time and the allocated total budgeted cost. Since different alternatives of possible durations and costs for the activities can be associated with a project and at the same time the existence of uncertainties in the project parameter,

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the project manager needs to optimized the project cost and duration simultaneously in the framework of fuzzy aspiration levels. These issues with proportional goal programming and fuzzy application have attracted the interest of more researchers and there are increasing papers dealing with these topics.Here fuzziness is used to improve the chances of success in project management. In addition, the degree of fuzziness not only deals with the lack of information but also supports the project managers that can make the wrong decisions in lower possibility. In another word, the experiences of the project managers on the project appropriate application reduces errors due to poor decisions may lead to opportunities for project failure.

The organization of this paper is as follows. Section 1 contain general concept of the problem area and Section 2 dedicates to the review of the literature and summary of the review of the literature. This section literally describe about the importance of the considered problem. In Section 3, the problem is introduced, and the notation and assumptions are defined at the same time, it presents computational experiments and describes the analysis of the results with various weight of the objectives as well as sensitivity analyses of different parameters to introduce the significant aspects of the model. In section 4, discussions are presented for overall result. Finally, in Section 5 the concluding remarks are given and future research directions are provided.

## 2. Literature Review

For project management decision analysis the most commonly used techniques are Gantt chart, Work Breakdown Structure, Milestone, Critical Path Method (CPM) and Project Evaluation and Review Technique (PERT). Considering the importance of time-cost optimization, various analytical and heuristic methods have been proposed by many researchers in recent decades including mathematical programming, algorithms and heuristics etc to solve PM decision problems. When any of these traditional techniques are used, however, related parameters are normally assumed to be deterministic/crisp (Lin and Gen 2007; Yin and Wang 2008, Al-Fanzine & Haouari, 2005) which is very ineffective for changing or uncertain environment. Because in real projects, time and cost of activities may face significant changes due to existing uncertainties such as inflation, economical and social stresses, labour performance, execution errors, design errors, natural events like climate changes and etc.

On the conventional techniques of PM, some modifications have been done by incorporating the concept of fuzzy logic. Liang and et al., (2004) has used fuzzy critical path method to improve fuzzy airport's ground operation decision analysis assuming fuzzy activity times as trapezoidal fuzzy number. Much researcher emphases on stochastic project management decision . Rabbani et al., (2007) developed a resource-constrained PM technique for stochastic networks resource allocation decisions having imprecise duration of each activity with a known distribution function in which the values of activities finish times were determined at decision points. But theoretical drawbacks of applying stochastic programming to PM decisions are lack of computational efficiency and inflexible probabilistic consideration. At the same time the conventional techniques of PM only concern about the cost and time trade-off but ignores the project crashing policy. Generally, the real PM decisions focus on the minimization of project completion time, and/or the minimization of total project costs through crashing or shortening duration of particular activities. The purpose of evaluating time-cost trade-offs is to develop a plan which the decision-maker (DM) can minimize the increase of project total cost and total crashing cost when shortening their total completion time. Liu (2003) formulated the critical path and the project crashing problems by linear programming with fuzzy activity times and then defuzzify the fuzzy activity times by ranking method. Again Jiuping et.al (2014) presented multimode resource-constrained project scheduling problem under fuzzy random environment by expectation method, where uncertainty is taken as a normal distribution.But any solution may be good if the decision maker is satisfied with the result. Various optimization technique may give optimum or near optimum result of a given PM problems. But this may fail to achieve by various situation. To deal with compromising solution Goal programming is so much effective. The goal programming technique is an analytical framework that a decision maker can use to provide optimal solutions to multiple and conflicting objectives. Arıkan (2001) and Fabiane*et.al* (2003) proposed a goal programming model for allocating time and cost in project management and manage the three projects with preemptive goals. Liang (2010) and Yang and Lin, (2013) focuses on developing a two-phase fuzzy mathematical programming approach for solving the multi-objective project management decision problems in a fuzzy environment. The model designed to minimizes objectives simultaneously by project managers in the framework of fuzzy aspiration levels enabling a decision maker to interactively modify the imprecise data and related parameters until a satisfactory solution is obtained. But here the weight of individual objectives is considered without any analysis. To solve time-cost trade-off problem, many researcher (Tamiz, 1998; Ghazanfari, 2008 and Liang, 2009) developed an approach by possibility goal programming with fuzzy decision variables. The model designed minimize simultaneously total projects costs, total completion time and crashing costs with reference to direct costs, indirect costs, contractual penalty costs, duration of activities, and the constraint of available budget. But possibility linear programming approach for an optimization problem with fuzzy parameters is possibilistic, which lead to the increase of the number of objectives function and constraints of the model. In the above goal programming and possibility goal programming method, the different membership functions are formulated from decision-maker preferences and experiences, but the decision-makers have the difficulties in making tradeoffs between the alternatives because of their inexperience and incomplete information. So there need some analytical way to define different membership functions.

Various types of heuristics have also been developed on the basis of the requirement of PM decision making incorporating with conventional techniques. Leu and et al., (2001); Lin and Gen (2007) incorporated fuzzy set theory with genetic algorithms to model uncertainty in time-cost trade-off problem. Abbasnia etal., (2008), Li et al.(2011*)* have investigated fuzzy logic based approach called Non-dominated Sorting Genetic Algorithm (NSGA) for time-cost trade-off problem in uncertain environment. This model cannot fully meet uncertainty of practical problem. Here, fuzzy set theory and Zimmermann's (1976) fuzzy programming technique have been developed into several fuzzy optimization methods to solve imprecise PM decision problems and avoiding unrealistic modeling in an uncertain environment. The minimum operator presented by Bellman and Zadeh (1970) is used to aggregate fuzzy sets, and the original MOLP problem is then converted into an equivalent ordinary LP form.Finally, in this paper, the imprecise nature of the input data is considered by implementing the interactive minimum operator method and weighted average method. The result obtained from minimum operators is used to determine the suitable membership function for goal programming method with weighted average aggregation and seek an efficient solution for it.

## 3. Methodology and Results

### 3.1 Problem description, assumptions and notations:

Assume that a project has n interrelated activities that must be executed in a certain order before the entire task can be completed in an uncertain environment. Accordingly, the incremental crashing costs for all activities, variable indirect cost per unit time and total budget are fuzzy. The original fuzzy MOLP model designed in this study aims to find out the minimum value of total project costs, total completion time and total crashing costs.

The following notation is used after reviewing the literature and considering practical situations [Liang (2010); Yang and Lin (2013)]. The proposed fuzzy mathematical programming model is based on the following assumptions:

(1) All of the objective functions are fuzzy with imprecise aspiration levels.
(2) All of the objective functions and constraints are linear equations.
(3) Direct costs increase linearly as the duration of activity is reduced from its normal time to its crash value.
(4) The normal time and shortest possible time for each activity and the cost of completing the activity in the normal time and crash time are certain.
(5) The available total budget is known over the planning horizons.
(6) The linear membership functions are adopted to specify fuzzy goals, and the minimum operator and weighted average operator are sequentially used to aggregate fuzzy sets.
(7) The total indirect costs can be divided into fixed costs and variable costs, and the variable costs per unit time are the same regardless of project completion time.

**Notations:**

| | |
|---|---|
| (I,j) | activity between events i and j, |
| g | index for objective function, for all g = 1,2,. . . ,K, |
| $Z_1$ | total project costs |
| $Z_2$ | total completion time |
| $Z_3$ | total crashing costs |
| $D_{ij}$ | normal time for activity (i,j) |
| $d_{ij}$ | minimum crashed time for activity (i,j) |
| $C_{Dij}$ | normal (direct) cost for activity (i,j) |
| $C_{dij}$ | minimum crashed (direct) cost for activity (i,j) |
| $k_{ij}$ | incremental crashing costs for activity (i,j) |
| $t_{ij}$ | crashed duration time for activity (i,j) |
| $Y_{ij}$ | crash time for activity (i,j) |
| Ei | earliest time for event i |
| $E_1$ | project start time |
| $E_n$ | project completion time |
| $T_{nc}$ | project completion time under normal conditions, |

| | |
|---|---|
| T | specified project completion time, |
| $C_l$ | fixed indirect costs under normal conditions, |
| m | variable indirect costs per unit time, |
| B | Available total budget. |

### 3.2 Fuzzy multi-objective linear programming model

In reality the project management activity is multi directional and multi objective type. Most of the project manager must consider minimizing total project costs, completion duration, crashing costs and contractual penalties, and/or maximizing profits and the utilization of equipment. Among these here three fuzzy objective functions are simultaneously considered during the formulation of the multi-objective PM decision model, as follows.

- Minimize total project costs

$$Min Z_1 = \sum_i \sum_j C_{D_{ij}} + \sum_i \sum_j \breve{K}_{ij} Y_{ij} + [C_l + \widetilde{m}(E_n - T_{nc})] \quad (1)$$

Here the terms, $\sum_i \sum_j C_{D_{ij}} + \sum_i \sum_j \breve{K}_{ij} Y_{ij}$ denote total direct costs including total normal cost and total crashing cost, obtained using additional direct resources such as overtime, personnel and equipment and the terms $[C_l + \widetilde{m}(E_n - T_{nc})]$ denote indirect cost including those of administration, depreciations, financial and other variable overhead cost that can be avoided by reducing total project time.. The symbol '~' represents fuzziness. Here $K_{ij}$ used to analysis the cost-time slopes for the various activities.

- Minimize total completion time

$$Min\, Z_2 = [E_n - E_1] \quad (2)$$

- Minimize total crashing costs

$$Min\, Z_3 = \sum_i \sum_j \breve{K}_{ij} Y_{ij} \quad (3)$$

**Constraints:**

- Constraints on the time between events i and j

$$E_i + T_{ij} - E_j \leq 0 \quad (4)$$

$$T_{ij} = D_{ij} - Y_{ij} \quad (5)$$

- Constraints on the crashing time for activity (i, j)

$$Y_{ij} \leq D_{ij} - d_{ij} \quad (6)$$

- Constraint on the total budget

$$Z_1 \leq \tilde{B} \tag{7}$$

- Non-negativity constraints on decision variables

$$t_{ij}, Y_{ij} and\ E_i \geq 0$$

### 3.3 Treatment of the fuzzy variable

This work assumes that the decision maker (DM) has already adopted the pattern of triangular possibility distribution to represent the fuzzy crashing cost, variable indirect costs per unit time and available total budget in the original fuzzy linear programming problem. In the process of defuzzification, this work applies Liou and Wang (1992) approach to convert fuzzy number into a crisp number.

If the minimum acceptable membership level α, then corresponding auxiliary crisp of triangular fuzzy number of $\widetilde{K_{ij}}$= $[\tilde{K}_{ij}^p, \tilde{K}_{ij}^m, \tilde{K}_{ij}^o]$ is:

$$\tilde{K}_{ij}^{\alpha} = \frac{1}{2}\left[\alpha \tilde{K}_{ij}^p + \tilde{K}_{ij}^m + (1-\alpha)\tilde{K}_{ij}^o\right] \tag{8}$$

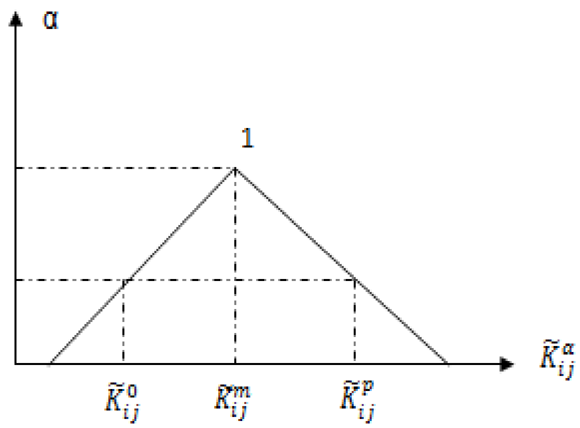


**Figure 1: Triangular Membership Function of $\tilde{K}_{ij}^{\alpha}$**

The primary advantages of the triangular fuzzy number are the simplicity and flexibility of the fuzzy arithmetic operations. For instance, Figure 1 shows the distribution of the triangular fuzzy number $k_{ij}$. In practical situations, the triangular distribution of $k_{ij}$may: (1) the most pessimistic value ($K_{ij}^p$) that has a very low likelihood; (2) the most likely value ($K_{ij}^m$) that definitely belongs to the set of available values; and (3) the most optimistic value ($K_{ij}^p$) that has a very low likelihood of belonging to the set of available values.

### 3.4 Problem Formulation Using Fuzzy Minimum Operator aggregation

Fuzzy set theory appears to be an ideal approach to deal with decision problems that are formulated as linear programming models with imprecision parameters. In this paper the net relative deviation is considered as fuzzy variable and converted into deterministic form using Zadeh's max-min operator concept as per Zimmermann (1978). Here, a linear membership function is defined by considering suitable upper and lower bounds to the objective function as given below.

First, the positive ideal solution (PIS) and negative ideal solution (NIS) for each of the fuzzy objective functions can be specified as follows:

$$Z_g^{PIS} = MinZ_g = lower\ limit \quad and \quad Z_g^{NIS} = MaxZ_g = upper\ limit$$

And then linear membership functions can be specified by requiring the DM to select the goal value interval $[Z_g^{PIS}$ and$Z_g^{NIS}]$. Accordingly, the corresponding, non-increasing continuous linear membership functions for the fuzzy objective functions can be expressed as follows:

$$f_g(Z_g) = \begin{cases} 1, Z_g \le Z_g^{PIS} \\ \frac{Z_g^{NIS} - Z_g}{Z_g^{NIS} - Z_g^{PIS}}, \quad Z_g^{PIS} \le Z_g \le Z_g^{PIS}, g = 1,2..k \\ 0 Z_g \ge Z_g^{NIS} \end{cases} \quad (9)$$

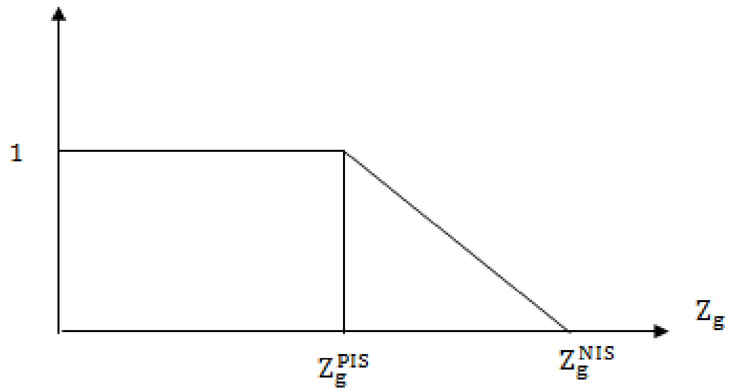


**Figure 2: Linear Membership Function of $Z_g$**

By introducing a min operator β an auxiliary variable, the equivalent fuzzy single goal linear programming problem is as follows:

Maximize β (0≤ β ≤ 1)

Subject to,

$$\beta \le \frac{Z_g^{NIS} - Z_g}{Z_g^{NIS} - Z_g^{PIS}} \quad (10)$$

And equation (4) – (7)

### 3.5 Goal Programming With Weighted Average Aggregation

In many important real world decision making situations, it may not be feasible, or desirable to try to reduce all goals of an organization. For example, rather than focusing only on maximizing profit or minimizing cost, the organization may be simultaneously be interested in maintaining all goals. Goal programming is an extension of linear programming with an additional feature of including conflicting objectives while still yielding a solution that is optimum with respect to the decision maker's specification of goal priorities. In goal programming, achievement of a set of goals, at some priority is always preferable to the achievement of a set of goals at a lower ranking priority. Accordingly, it is possible to include several weighted goals within each ranking. Then the fuzzy goal programming approach is used to achieve the highest degree of each of the membership goals and thereby obtain the most satisfactory solution for all decision makers in a multi-objective problem. By introducing the auxiliary variable β, the fuzzy goal programming problem can be converted into an equivalent ordinary LP model by the weighted average aggregation as follows,

$$\mathrm{Max}\ \beta = \sum_{g=1}^{k} W_g\, \beta_g \tag{11}$$

Subject to,

$$\beta \leq \frac{Z_g^{NIS} - Z_g}{Z_g^{NIS} - Z_g^{PIS}} \tag{12}$$

$$\sum_{g=1}^{k} W_g = 1 \tag{13}$$

$$0 \leq W_g \leq 1 \tag{14}$$

Equation (4) – (7)

Where $W_g$ (g = 1,2, . . . ,K) is the corresponding weight of the $g^{th}$ fuzzy objective function chosen by DM.

### 3.6 Solutions Procedure

Step1. Formulate the original fuzzy MOLP model for the project management problems according to Eqs. (1) – (7).
Step2. Provide the minimum acceptable membership level, α and then convert the fuzzy variable into crisp ones using the fuzzy ranking number method according to Eqs. (8).
Step3. Specify the degree of membership $f_g(Z_g)$ for several values of each objective function $Z_g$, g = 1, 2,3 by PIS and NIS.
Step4. Introduce the auxiliary variable β, thus enabling aggregation of the original fuzzy MOLP problem into an equivalent ordinary single-objective LP form using the minimum operator method.
Step5. Solve the ordinary LP problem by MATLAB. If the DM is dissatisfied with the initial solutions, the model should be adjusted until a preferred satisfactory solution is obtained.
Step6. Determine the goal value of each objective.

Step7. Formulate the problem according to the goal programming approach with weighted average aggregation and solve original fuzzy MOLP problem into an equivalent ordinary single goal LP form by genetic algorithm.

### 3.7 Data description

Daya Technologies Corporation was used as an industrial case study to demonstrate the practicality of the developed methodology (Wang and Liang 2004; Liang 2009). Daya is the world's first ball screw manufacturer certified to ISO 9001, ISO 14001, and OHSAS18001, and is also the major manufacturer producing the super precision ball screw, linear stage, guide ways, linear bearing and aerospace parts in Taiwan. Its products are mainly distributed throughout Asia, North America and Europe. Table: 1 lists the basic data of the Daya case.

**Table 1: Summarized Data in the Daya Case (in US Dollar)**

| (i,j) | $D_{ij}$day | $d_{ij}$day | $C_{D_{ij}}$\$ | $C_{d_{ij}}$\$ | $K_{ij}$\$/day |
|---|---|---|---|---|---|
| 1-2 | 14 | 10 | 1000 | 1600 | (132,150, ,162) |
| **1-5** | 18 | 15 | 4000 | 4540 | (164, 180, 198) |
| 2-3 | 19 | 19 | 1200 | 1200 | - |
| 2-4 | 15 | 13 | 200 | 440 | (112, 120, 128) |
| 4-7 | 8 | 8 | 600 | 600 | - |
| 4-10 | 19 | 16 | 2100 | 2490 | (112, 130, 140) |
| 5-6 | 22 | 20 | 4000 | 4600 | (280, 300, 324) |
| 5-8 | 24 | 24 | 1200 | 1200 | - |
| **6-7** | 27 | 24 | 5000 | 5450 | (136, 150, 166) |
| 7-9 | 20 | 16 | 2000 | 2200 | (34, 50, 58) |
| 8-9 | 22 | 18 | 1400 | 1900 | (111, 125, 139) |
| **9-10** | 18 | 18 | 700 | 1150 | (120, 150, 160) |
| **10-11** | 20 | 18 | 1000 | 1200 | (80, 100, 108) |

Other relevant data are as follows: fixed indirect costs \$12000, saved daily variable indirect costs (\$144, \$150, \$154), available budget (\$36,000, \$38,000, \$43,000), and project duration under normal conditions 125 days. The project start time is set to zero. The minimal acceptable possibility for all imprecise numbers is specified as 0.5. The critical path is 1–5–6–7–9–10–11.

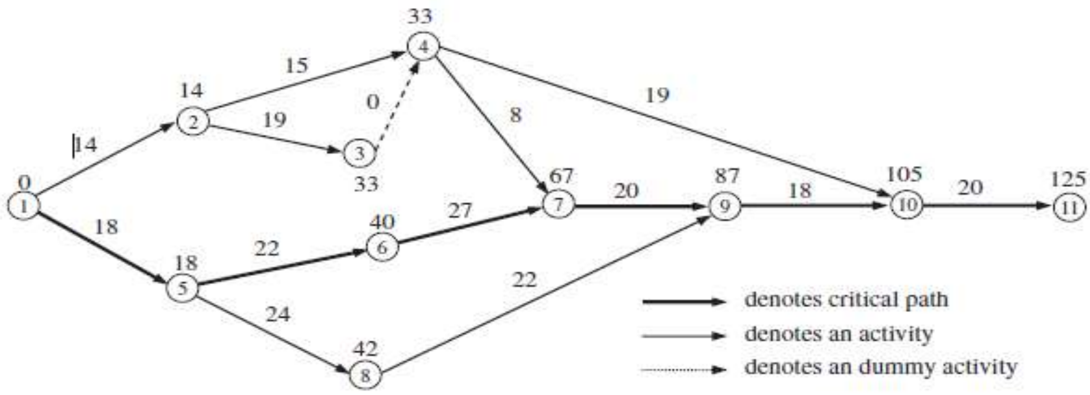


**Figure 3: The Project Network of the Case**

### 3.8 Computational experiments

First, formulate the original fuzzy MOLP model for solving the multi objective project management problem according to equation (1) – (7) and solve the multi objective project management problem using the ordinary single-objective LP problem by MATLAB computer software to obtain the initial solutions for each of the objective functions to determine $Z_1^{PIS} and Z_1^{NIS}$ for objective $Z_1$, $Z_2^{PIS} and Z_2^{NIS}$ for objective $Z_2$ and $Z_3^{PIS} and Z_3^{NIS}$ for objective $Z_3$. After running the program by MATLAB computer software the result shown in Table: 2. The result is obtained for minimum acceptable membership level α = 0.5**.**

**Table 2: PIS and NIS**

| Objective function | PIS | NIS |
|---|---|---|
| $Z_1$ | 35500 | 36400 |
| $Z_2$ | 110 | 125 |
| $Z_3$ | 0 | 3701 |

After finding this NIS and PIS, the linear membership functions of each objective function is defined according to the equation (9). Then the problem presented in equation (10) and equations (4) – (7) is formulated by using fuzzy minimum operator approach as follows,

Maximize β (0≤ β ≤ 1)

Subject to the

$$\beta \leq \frac{36400 - Z_1}{36400 - 35500} \quad (15)$$

$$\beta \leq \frac{125 - Z_2}{125 - 110} \quad (16)$$

$$\beta \leq \frac{3701 - Z_3}{3701 - 0} \quad (17)$$

And equation (4) – (7)

After transforming multi-objective linear programming to single objective linear programming by minimum operator method the problem is solved by MATALB software. The optimal solutions are $Z_1$=$35815, $Z_2$= 115.3 days and $Z_3$=$1295 and the overall level of satisfaction of decision maker with the given objective values is 0.65 that is shown in table: 4.

### 3.9 Goal Defining for Goal Programming Method

Each objective is not equally important to the decision maker or to the case for various situations. So he result can be modified by expectation of decision maker that is how the weight is given by decision maker to the each objective. The value of the relative weights among of multiple goals can be adjusted subjectively based on the DM's experience and knowledge.

The relative importance of each objective function is different in each project case-by-case and is dependent on the company's perception. In practice, the relative weights of the objective could be determined by referring to the previous project's records or by asking the DM to subjectively give his/her opinion about the importance of the goal of the project. It is also possible that the DM uses a systematic technique like (Analytical Hierarchy Process) AHP (Suo et al., 2012; Saaty, 2008; Chien and Barthorpe, 2013 and Al-Harbi, 2001) or Analytical Network Process (ANP) to specify the relative weights of the objectives. Here, we assume that, based on the project specifications and the goals of the company, the decision maker adopts the priority of the objective functions as $w1 > w2 > w3$ and set the objective's weight vector equal to W= (0.37, 0.34, 0.29).
Then, the goal programming model is formulated by using the above weight where, the positive ideal solution (PIS) and negative ideal solution (NIS) for each of the fuzzy objective functions are taken as lower limit and upper limit respectively. Then the problem presented in equations (11) – (14) and (4) – (7) is formulated as auxiliary single objective goal programming based on the weighted average aggregation and linear membership function defined in Eqs. (9) as follows:

$$\text{Max}\,\beta = 0.37 * \beta_1 + 0.34 * \beta_2 + 0.29 * \beta_3 \tag{18}$$

Subject to the

$$\beta \leq \frac{36400 - Z_1}{36400 - 35500} \tag{19}$$

$$\beta \leq \frac{125 - Z_2}{125 - 110} \tag{20}$$

$$\beta \leq \frac{3701 - Z_3}{3701 - 0} \tag{21}$$

$$W_1 + W_2 + W_3 = 1 \tag{22}$$

$$0 \leq W_g \leq 1 \tag{23}$$

Equation (4) – (7)

Where $W_g$ (g = 1,2, . . . ,K) is the corresponding weight of the $g^{th}$ fuzzy objective function chosen by DM.

### 3.9.1 Goal Programming with Genetic Algorithm

The proposed goal programming model is solve by genetic algorithm. The genetic algorithm works with an initial population and each initial population consists of set of initial solutions, represented by chromosomes. Each individual solution or chromosome carries encoded information represented by genes that are assigned to variables. GA works interactively and improves the population of solutions using three search operators which are selection, crossover, and mutation. The selection operator selects the individuals as parents and allows to produces children for the next generation. It gives the preference to the fittest individuals. The crossover operator, a genetic operator generates new chromosomes by combining pairs of existing potential and fitter chromosomes to be let them playing role of parents to produce the next generations. Mutation operator is also a genetic operator that maintains the diversity by making small changes on genes of individual solutions. The process continues until the stopping criteria are met. The search parameters of the multi-objective genetic algorithm have been set at random and for various GA parameters, each objective value with goal value is given in table 3 below.

Table 3: GA Search parameters setting and their corresponding output

| GA Parameters for goal programming / Options | Scenario 1 | Scenario 2 | Scenario 3 | Scenario 4 |
|---|---|---|---|---|
| Population<br>• Type<br>• Population Size<br>• Creation Function | Double<br>300<br>Feasible Population | Double<br>300<br>Constraint dependent | Double<br>300<br>Feasible Population | Double<br>300<br>Constraint dependent |
| Fitness scaling<br>• Function | Proportional | Rank | Rank | Rank |
| Selection<br>• Function<br>• size | Tournament<br>3 | Tournament<br>4 | Stochastic Uniform | Roulette wheel |
| Reproduction<br>• fraction | 0.7 | 0.8 | 0.8 | 0.8 |
| Mutation function | Adaptive feasibility | Constraint dependent | Adaptive feasibility | Constraint dependent |
| Crossover<br>• function<br>• ratio | Heuristic<br>1.2 | Intermediate<br>0.8 | Two point | Scattered |
| Migration direction | Both (0.2) | Forward (0.2) | Both (0.2) | Both (0.2) |
| Pareto front population fraction | 0.35 | 0.35 | 0.35 | 0.35 |
| Number of iteration required | 153 | 121 | 231 | 148 |
| Value Multi-objective function<br>• $\beta$<br>• $Z_1$<br>• $Z_2$<br>• $Z_3$ | 0.674<br>$35793<br>114.8 days<br>$1206 | 0.733<br>$35740<br>111 days<br>$999.5 | 0.7410<br>$35731<br>112 days<br>$962 | 0.728<br>$35735<br>111 days<br>$994 |

The mathematical representation of the goal programming approach with weighted average aggregation given above where the positive ideal solution (PIS) and negative ideal solution (NIS) for each of the fuzzy objective functions are taken as lower limit and upper limit respectively and genetic algorithm is used to solve the ordinary LP model and the result shown in table 4.The proposed Goal Programming model provides the overall levels of DM satisfaction ($\beta$ value) that gives the multiple fuzzy goal values ($Z_1$, $Z_2$, and $Z_3$). If the solution is $\beta$ =1, then each goal is fully satisfied; if $0 < \beta < 1$, then all of the goals are satisfied at the level of $\beta$, and if $\beta$ =0, then none of the goals are satisfied. For example, the overall degree of DM satisfaction (β) with the goal values ($Z_1$=$35815, $Z_2$= 115.25 days and $Z_3$=$1295) was initially generated as 0.65. At the same time if the decision maker weight the three objective functions by 0.37, 0.34 and 0.29 respectively then for the minimum acceptable membership level α=0.5, the overall degree of DM satisfaction (β) is 0.74 with the goal values $Z_1$=$35731, $Z_2$= 112 days and $Z_3$=$962

**Table 4: Result of Two Methods**

| Item | solution of MOLP model with min operator aggregation method | Improved solution of fuzzy goal programming with weighted average aggregation by genetic algorithm |
|---|---|---|
| Goal values | β =0.65<br>$Z_1$=$35815, $Z_2$= 115.3 days and $Z_3$=$1295 | β = 0.74<br>$Z_1$=$35731, $Z_2$= 112 days and $Z_3$=$962 |
| $Y_{ij}$days) | $Y_{12}$=0, $Y_{15}$= 0.67, $Y_{23}$=0 $Y_{24}$=0 $Y_{47}$=0 $Y_{410}$=0 $Y_{56}$=0 $Y_{58}$=0, $Y_{67}$=3, $Y_{79}$=4, $Y_{89}$=0, $Y_{910}$=0, $Y_{1011}$=2 | $Y_{12}$=4, $Y_{15}$= 2.9, $Y_{23}$=0 $Y_{24}$=1.75 $Y_{47}$=0 $Y_{410}$=1 $Y_{56}$=1.92 $Y_{58}$=0, $Y_{67}$=2.93,$Y_{79}$=4, $Y_{89}$=3.76, $Y_{910}$=2.7, $Y_{1011}$=1 |
| $t_{ij}$(days) | $t_{12}$=14, $t_{15}$=17.33, $t_{23}$=19, $t_{24}$=15, $t_{47}$=8, $t_{410}$=19, $t_{56}$=22, $t_{58}$=24, $t_{67}$=24, $t_{79}$= 16<br>$t_{89}$=22, $t_{910}$=18, $t_{1011}$=18 | $t_{12}$=10, $t_{15}$=15, $t_{23}$=19 , $t_{24}$=13.2, $t_{47}$=8 $t_{410}$=18, $t_{56}$=20 , $t_{58}$=24, $t_{67}$=24, $t_{79}$= 16 , $t_{89}$=18, $t_{910}$=18, $t_{1011}$=19 |
| $E_{ij}$(days) | $E_1$=0 ,$E_2$=14, $E_3$=33, $E_4$=29, $E_5$=17.33, $E_6$=39.33, $E_7$=63.33, $E_8$=41.3, $E_9$=79.3, $E_{10}$=97.3, $E_{11}$=115.3 | $E_1$=0 , $E_2$=10, $E_3$=29, $E_4$=23, $E_5$=15, $E_6$=35, $E_7$=59, $E_8$=56.8, $E_9$=75, $E_{10}$=93, $E_{11}$=112 |

## 4. Discussion

Form table 5, it is seen that a fair difference and interaction exists in the trade-offs and conflicts among dependent objective functions. For instance, the combination of the total project costs and completion time in scenario1 of table 5 is $Z_1$=$35500 and $Z_2$ =110 days with β value 1.0. Again the combination of the completion time and the crashing cost was $Z_2$ = 115.5 days and $Z_3$ = $1298 with β value 0.64.

Table 5: Result analyses with two objectives and removing one objective simultaneously

| Item | Scenario 1 | Scenario 2 | Scenario 3 |
|---|---|---|---|
| β Value | 1 | 1 | 0.64 |
| $Z_1$ | 35500 | 35500 | - |
| $Z_2$ | 110 | - | 115.5 |
| $Z_3$ | - | 0 | 1298 |

So, the proposed method yields an efficient compromise solution. Generally, the β value may be adjusted to identify better results if the DM did not accept the initial overall degree of this satisfaction value. Additionally, the optimal solution yielded by the minimum operator method may not be an efficient solution, and the computational efficiency of the solution is not been assured. The minimum operator is preferable when a DM wants to make values of the optimal membership functions approximately equal or when a DM believes that the minimum operator is an approximate representation. To overcome the disadvantage of using the minimum operator, the compensatory weighted average aggregation is employed for to obtain overall decision maker satisfaction degree. At the same time the interactive change of the search parameters of the GA provide better result for the proposed goal programming model. Accordingly, the proposed model meets the requirements of the practical application since it can simultaneously minimize the total production costs, total distribution costs, and total distribution time.

## 5. Conclusion

This research focused on realistic and flexible project management decision based on fuzzy goal programming with weighted average aggregation. The above mentioned fuzzy goal programming method for project management considers multiple conflicting objectives under uncertain environment. The model also offers the interactive change of the search parameters of the GA in order to tuning the desired results. Here minimum operator method is used to aggregate all fuzzy set at first and fuzzy goal programming method is develop with respect to the results obtained from minimum operator method. The flexibility of decision maker is one of the important parameter of this work and this is the ability to identify optimal value by interactively. The proposed method helps decision makers to choose the desired compromise solution for time-cost trade-off within a time limit under different risk levels that varies with respect to the minimum acceptable level. Here the problem considers completion time in a suitable range for multi-objective project management (PM) decisions and this time is balanced with respect to the project crashing cost. The proposed model simultaneously minimize the total project costs, crashing cost and the total time with reference to fuzzy data in the framework of satisfaction level of decision maker interactively.

The developed model could also be further extended by adopting systematic approaches, such as Analytical Network Process (ANP) and other types of fuzzy membership functions like flexible membership function and dynamic membership function.

## References

Abbasnia, R., Afshar, A., and Eshtehardian, E., 2008, 'Time - cost trade-off problem in construction project management, based on fuzzy logic', *Journal of Applied Sciences, Vol. 8, No. 22, Pp.4159-4165.*

Al-Fawzan, M.A. and Haouari, M., 2005,'A bi-objective model for robust resource-constrained project scheduling', *International Journal of production economics, Vol. 96 No.2, pp.175-187.*

Al-Harbi, K.M.A.S., 2001,'Application of the AHP in project management', *International journal of project management, Vol. 19, No. 1, pp.19-27.*

Arıkan, F. and Güngör, Z., 2001,'An application of fuzzy goal programming to a multi-objective project network problem', *Fuzzy sets and systems, Vol. 119, No.1, pp.49-58.*

Bellman, R.E. and Zadeh, L.A., 1970,'Decision-making in a fuzzy environment', *Management science, Vol. 17, No. 4, pp.B-141.*

Chien, H.J. and Barthorpe, S., 2013,'Using Analytic Hierarchy Process to analyse the critical success factors for performance management of the Taiwanese Veterans Home', *International Journal of Applied*, Vol. *3, No.* 7, Pp.48-63

Chanas, S. and Zieliński, P., 2001,'Critical path analysis in the network with fuzzy activity times', *Fuzzy sets and systems, Vol. 122, No. 2, pp.195-204*.

De Oliveira, F., Volpi, N.M.P. and Sanquetta, C.R., 2003,'Goal programming in a planning problem', *Applied Mathematics and Computation, Vol. 140, No. 1, pp.165-178.*

Ghazanfari, M., Yousefli, A., Ameli, M.J. and Bozorgi-Amiri, A., 2009,'A new approach to solve time–cost trade-off problem with fuzzy decision variables', *The International Journal of Advanced Manufacturing Technology, Vol. 42, No. 3-4, pp.408-414.*

Leu, S.S., Chen, A.T. and Yang, C.H., 2001,'A GA-based fuzzy optimal model for construction time–cost trade-off', *International Journal of Project Management, Vol. 19, No. 1, pp.47-58.*

*Li, H., Hussein, M. A., & Lei, Z. (2011). Incentive genetic algorithm based time–cost trade-off analysis across a build–operate–transfer project concession period. Can. J. Civ. Engrg., 38, 166-174.*

Liu, S.T., 2003,'Fuzzy activity times in critical path and project crashing problems',*Cybernetics &Systems, Vol. 34, No. 2, pp.161-172.*

Lin, C.M. and Gen, M., 2007,'Multiobjective resource allocation problem by multistage decision-based hybrid genetic algorithm', *Applied Mathematics and Computation*, Vol. *187, No.* 2, pp.574-583.

Liang, G.S. and Han, T.C., 2004,'Fuzzy critical path for project network,' *Information and Management Sciences, Vol. 15, No. 4, pp.29-40.*

Liang, T.F., 2009,'Application of fuzzy sets to multi-objective project management decisions', *International Journal of General Systems, Vol. 38, No. 3, pp.311-330.*

Liang, T.F., 2010,'Applying fuzzy goal programming to project management decisions with multiple goals in uncertain environments', *Expert Systems with Applications, Vol. 37, No. 12, pp.8499-8507.*

Liou, T.S. and Wang, M.J.J., 1992,'Ranking fuzzy numbers with integral value', *Fuzzy sets and systems, Vol. 50, No. 3, pp.247-255.*

Rabbani, M., Ghomi, S.F., Jolai, F. and Lahiji, N.S., 2007,'A new heuristic for resource-constrained project scheduling in stochastic networks using critical chain concept', *European journal of operational research, Vol. 176, No. 2, pp.794-808.*

Saaty, T.L., 2008, 'Decision making with the analytic hierarchy process', *International journal of services sciences, Vol. 1, No. 1, pp.83-98.*

Suo, M.Q., Li, Y.P. and Huang, G.H., 2012,'Multicriteria decision making under uncertainty: An advanced ordered weighted averaging operator for planning electric power systems', *Engineering applications of artificial intelligence, Vol. 25, No. 1, pp.72-81.*

Tamiz, M., Jones, DF, and Romero C.,1998,'Goal programming for decision making: an overview of the current state-of-the-art', *European Journal of Operational Research, Vol. 111, Pp. 569–81.*

Xu, J. and Feng, C., 2014,'Multi-mode resource-constrained multiple project scheduling problem under fuzzy random environment and its application to a large scale hydropower construction project', *The Scientific World Journal*, Volume 2014, Article ID 463692, 20 pages

Yang, M.F. and Lin, Y., 2013,' Applying fuzzy multi-objective linear programming to project management decisions with the interactive two-phase method', *Computers & Industrial Engineering, Vol. 66, No. 4, pp.1061-1069.*

Yin, P.Y. and Wang, J.Y., 2008,‘ Optimal multiple-objective resource allocation using hybrid particle swarm optimization and adaptive resource bounds technique’, *Journal of Computational and Applied Mathematics, Vol. 216, No. 1, pp.73-86.*

Zadeh, L., 1978, Fuzzy sets as basis for a theory of possibility, *Fuzzy Sets and Systems*, Vol. 1, 3-28

Zimmermann, H.J., 1978, ‘Fuzzy programming and linear programming with several objective function’, *Fuzzy sets and systems, Vol. 1, No. 1, pp.45-55.*