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A Mixed Integer Linear Programming Model for Solving Closed Loop Supply Chain Problems

Aninda Saha, Md. Asadujjaman* and Md. Asaduzzaman

Design of supply chain network considering both forward and reverse flows has been increasing day by day due to resource constraints, increased costs and the importance of utilizing returned products. This paper attempts to integrate both forward and reverse logistics to design a general closed loop supply chain (CLSC) network consisting of manufacturing plant, distribution center and customer market. In this paper, a single-product multi-period mixed integer linear programming model has been designed to minimize the costs of CLSC and to optimize the facility location along with network flows. To evaluate the efficiency of the designed model, it has been implemented to a real case study on a cement industry. Branch and Bound algorithm is used to determine the optimal solution. The result shows that the designed model has successfully optimized the location of facilities and network flows. The result indicates that, in practice, the proposed model is applicable.

Keywords: Closed Loop Supply Chain, Mixed Integer Linear Programming, Network Flow.

1. Introduction

The concept of closed-loop supply chain (CLSC), which means the combination of forward and reverse logistics (Guide et al 2013), is now widely taking attention as a result of the recognition that both logistics have strong influence on supply chain performance and it is needed to manage these logistics simultaneously. Lee and Dong (2008) presented a research work which showed that the reverse logistics network has a strong influence on the forward logistics and vice versa, therefore, the design of the forward and reverse logistics network is needed to be integrated. CLSC problems have been important issues for both researchers and practitioners from the last decade because of the customer expectations, increasing business competition, and regulatory pressures. Contrasting with reverse logistics, closed loop supply chain involves not only the reverse flows of the product or product parts from the end users (external customers) to the manufacturers or related facilities such as distribution centers, but also the forward flows of raw materials and goods from suppliers to manufacturers and then to customers.

Numerous studies have been conducted on supply chain forward logistics (Dharky and Bangar 2013; Parida and Andhare 2014; Billal et al 2015; Chukwuma and

Aninda Saha, Department of Industrial & Production Engineering, Rajshahi University of Engineering & Technology, Rajshahi-6204, Bangladesh, E-mail: aninda.ndc@gmail.com
*Md. Asadujjaman,Corresponding Author, Department of Industrial & Production Engineering, Rajshahi University of Engineering & Technology, Rajshahi-6204, Bangladesh,
E-mail: jonikhan007@yahoo.com
Md. Asaduzzaman, Department of Industrial & Production Engineering, Rajshahi University of Engineering & Technology, Rajshahi-6204, Bangladesh, E-mail: m.asadie111@gmail.com

Chukwuma 2015; Sahu and Victor 2016) as well as reverse logistics (Min et al 2006; Shi et al 2009; Hu and Sheu 2013; Roghanian and Pazhoheshfar 2014). Chukwuma and Chukwuma (2015); Sahu and Victor (2016) have minimized the supply chain transportation cost for cement manufacturing industries that have considered only forward logistics. Billal et al (2015) have shown that the optimal supply chain network opportunity considering forward logistics in case of Bangladesh increases customer satisfaction with minimum transportation cost. Shi et al (2009) have developed a mixed integer linear programming model for minimizing total costs of reverse logistics including transportation cost, fixed cost of operating, costs of collecting and processing centers, and operation costs of facilities to deal with medical waste returned from hospitals to a manufacturer. A multi-objective post-disaster debris reverse logistics model has been developed by Hu and Sheu (2013) where environmental and operational risks and psychological cost have been considered for minimizing the total reverse logistics cost. Gou et al (2008) discussed an open-loop reverse chain, including a single, centralized return center (CRC) and multiple local collection points (LCPs). This study has extended their research work to find out the optimal economic delivery batch size for the LCPs as well as the optimal handling batch size for the CRC in order to minimize the long-run average cost for the open-loop reverse supply chain.

A closed-loop supply chain (CLSC) has stretched out the traditional concept of supply chains by smoothly exploring the coordination between the forward flows and reverse flows. Industries, which get advantages from effective reverse logistics, increase their profits by decreasing investment in resources, as well as the reduction of costs. Thus, CLSC involves problems associated with re-manufactured products that create an additional challenge for the design and planning problem. Therefore, experts and academics are paying an increasing interest to CLSCs, directing to collect and recycle returned products. Some literature exists that considers both forward and reverse logistics (Üster et al 2007; Saffari et al 2015; Ene and Öztürk 2014). Fleischmann et al (2001) developed a generic model for the design of CLSC logistics networks. The forward flow had been considered jointly with the reverse flow in their model, allowing the simultaneous definition of the optimal distribution and recovery networks. A MILP formulation was proposed that constitutes an extension of the traditional warehouse location problem. This model was used to test of its performance by two previously published case studies and a study on the benefit of integrating both chains was performed. Jayaraman et al (2003) proposed a solution method to solve the single product two-level hierarchical location problem involving both the forward and reverse supply chain operations of hazardous products. After that, their research methodology had been extended into a heuristic to handle relatively large sized problems. Amaro and Barbosa-Póvoa (2007) proposed a closed-loop problem, which was an analysis of continuous and discrete formulation models using the standard branch and bound method, however, it was not so effective in designing stage decisions. Üster et al (2007) have designed a closed loop supply chain network in which the forward network flow was existed and only the collection and recovery centers was considered to be located.

Das and Chowdhury (2012) presented a mixed integer linear programming model for a CLSC planning to maximize the overall profit considering the modular design, architecture and different quality levels of sold products on the markets. Georgiadis et al (2006) considered a single supplier CLSC logistic network that tries to use recycled materials as its principal inputs of raw materials. It presented a new decision making method for capacity planning based on the balanced tradeoff between profit and capacity utilization and the dynamic periodic review approach. The presented methodology tried to integrate capacity planning into a system dynamics model. Ene and Öztürk (2014) presented a mathematical model for multi-stage and multi-period open loop supply chain (OLSC) network, which maximized total profit of the network and determined facility locations and material flows between stages in each period. Saffari et al (2015) have developed a mathematical model for single-product and single-period to minimize the total cost of a closed loop supply chain. In this research, a mathematical formulation has been developed to solve single-product, multi-period problems instead of single-period closed-loop supply chain problems, related to the model developed by Saffari et al (2015) and applied to a large scale problem.

The main contribution of this paper may be described in three points. First, a general structure of a closed loop supply chain has been defined. Second, this research constructs the formulations for minimizing the total supply chain cost. Finally, this paper shows how to optimize facility locations along with network flows. In consequence, this research will be helpful for the supply chain managers to minimize the total supply chain cost with optimizing the network considering both forward and reverse flow of the products.

The research presented in this paper aims to design a closed-loop supply chain network with two kinds of facilities containing manufacturing plant and distribution centers for forward flow; whereas, for reverse flow, it has been considered that the defective or non-conforming products are returned to distribution centers from customers. In this problem, distribution centers have also been considered as collection centers. In the mathematical model, objective functions have been designed to locate facilities optimally along with optimizing network flows.

The rest of this paper is organized as follows. Section 2 describes the proposed closed loop supply chain model. In section 3, we illustrate the proposed model for a real life example. Finally, section 4 provides conclusions and suggestions for future work.

2. The Proposed Closed Loop Supply Chain Model

2.1 Assumptions

The assumptions of the model are:

1. There is only one product in the network.
2. Manufacturing plants and customer locations are fixed.
3. Demand is deterministic.
4. Capacity of facilities is finite and deterministic.
5. All the returned products are collected to the distribution centers.
6. Raw material is assumed to be infinite.
7. Quality of collected products from different spots is the same.
8. The percent of returned products is determined for the manufacturing plants.
9. Network flows' capacity is considered to be infinite.

2.2 Model Description

The following notations are employed for formulating the CLSC problem:

2.2.1 Indices

j index of fixed locations of manufacturing plants, $j = (1,2,3,\ldots,J)$

k index of potential distribution centers, $k = (1,2,3,\ldots, K)$

l index of potential market locations, $l = (1,2,3,\ldots, L)$

t index of planning periods, $t = (1,2,3,\ldots,T)$

2.2.2 Parameters

The following parameters are considered in formulating the CLSC problem:

g_k fixed cost of opening distribution center *k*
CZ_k capacity of distribution center *k*
JC_j cost of manufacturing final product in plant *j*
KC_k operational cost of each product in distribution center *k*
μ_{ab} transportation cost of each unit product between facilities *a, b*
f_l return rate of products in market *l*
d_l demand quantity in market *l*
cv_j quantity of raw materials received by manufacturing plants *j*

2.2.3 Decision Variables

The decision variables are:
z_k if distribution center is located in spot *k* at period *t*, 1 or 0

B_{tjk} product quantity shipped from final product plant *j* to distribution center *k* at time period *t*

Y_{tkl} product quantity shipped from distribution center *k* to market *l* at time period *t*

Q_{tlk} product quantity shipped from demand market *l* to distribution center *k* at time period *t*

Q_{tkj} product quantity shipped distribution center *k* to manufacturing plant *j* at time period *t*

2.3 Objective Function

The objective function for the proposed closed loop supply chain is as follows:
Min Z (Total Cost) = $\sum_k g_k z_k$ (fixed cost of distribution center) + $\sum_t \sum_j \sum_k B_{tjk}(\mu_{jk} + JC_j)$ (operational cost of manufacturing plant + transportation cost

from j to k) + $\sum_t\sum_k\sum_l Y_{tkl}(\mu_{kl}+KC_k)$ (operational cost of distribution centers + transportation cost from k to l) + $\sum_t\sum_l\sum_k Q_{tlk}\mu_{lk}$ (transportation cost from I to k) + $\sum_t\sum_k\sum_j Q_{tkj}\mu_{kj}$ (transportation cost from k to j) (1)

2.3.1 Subject to

$\sum_t\sum_k Y_{tkl} = d_l$ (Products shifted from the distribution center to satisfy demand in customer market l) (2)

$\sum_t\sum_k Q_{tjk} = \sum_t\sum_k Y_{tkl} f_l$ (All returned products from demand markets are collected in distribution centers) (3)

$\sum_t\sum_j B_{tjk} = \sum_t\sum_l Y_{tkl}$ (Product quantity of j to k equals k to l) (4)

$\sum_t\sum_j B_{tjk} \le \sum_t\sum_k Q_{tkj} + CV_j$ (Equilibrium condition in manufacturing plant for reverse flow) (5)

$\sum_t\sum_k Q_{tkj} \le \sum_t\sum_k Q_{tlk}$ (Equilibrium condition in distribution center for reverse flow) (6)

$\sum_t\sum_l Y_{tkl} \le CZ_k Z_k$ (Ensuring capacity of distribution centers) (7)

$\sum_t\sum_k Q_{tkj} \le CZ_k Z_k$ (Ensuring capacity of distribution centers for reverse flow) (8)

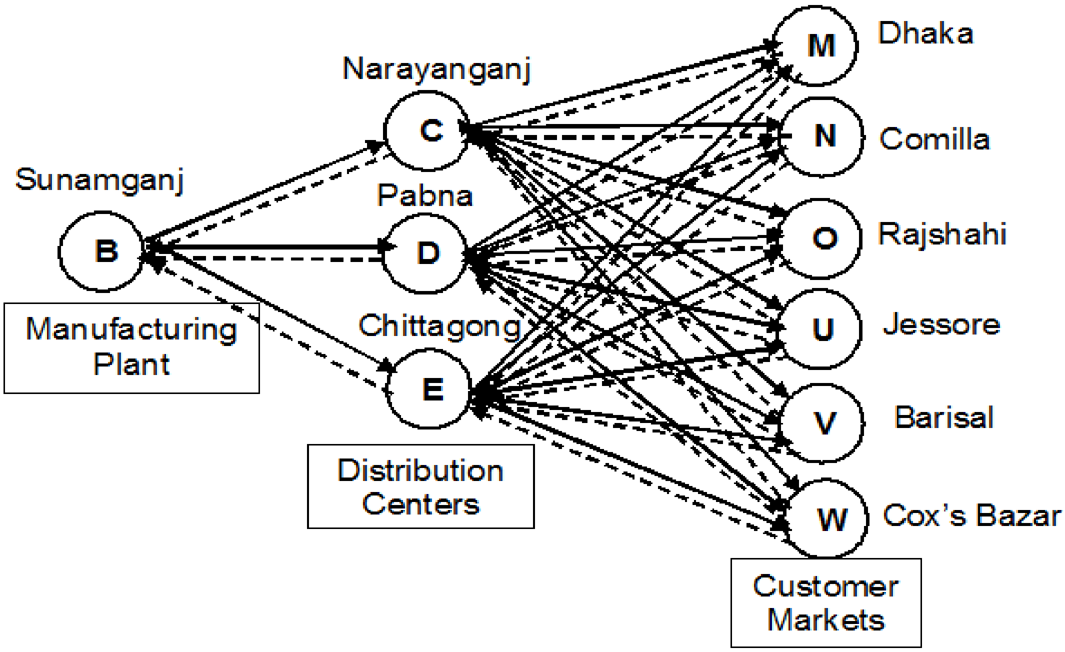
$Zc, Zd, Ze \in \{0,1\}$ (Binary conditions for distribution centers) (9)

$B_{tjk}, Y_{tkl}, Q_{tlk}, Q_{tkj} \ge 0$ (Non-negativity constraints) (10)

3. Case Study and Analysis of Result

We have performed a case study in a cement industry. The model is designed for a single plant with 3 distribution centers and 6 major customer market locations and also for 3 planning periods (4 months per period). In this network, the received raw materials along with the returned products were utilized for manufacturing cement bags in a manufacturing plant. Therefore, the produced cement bags were shipped to customer markets through distribution centers.

Figure 1: Closed Loop Supply Chain Network



The defective products were collected to distribution centers, which were sent and recycled in the manufacturing plant. The closed loop supply chain network for the case study is illustrated in figure 1. Table 1 shows the data on product quantity shifted among facilities of the industry for the year of 2015, Table 2 shows the corresponding fixed costs, operating costs and the capacities of the facilities.

Table 1: Product Quantity Data

	1st Period (January to April)	2nd Period (May to August)	3rd Period (September to December)
Final Manufacturer Plant to Distribution center (in bags)	412671	413262	412962
Distribution Center to Customer Market (in bags)	402116	402516	402318
Customer Market to Distribute Center (in bags)	830	828	852

Table 2: Cost and Capacity Data

	Manufacturing Plant	Distribution Centers		
	Sunamganj	Narayanganj	Pabna	Chittagong
Fixed or Setup Cost (Tk)	2,200,837,497	1,856,475	1,216,542	1,572,865
Operational Cost per bag (Tk)	12	3	3	3
Capacity (MT)	1,500,000	600,000	550,000	530,000

The numerical example has been solved by branch and bound (B&B) algorithm using LINGO 16.0 software. It takes 0.06 second and 52 iterations to solve the example problem. Therefore, the proposed mixed integer linear programming model is computationally very efficient. The optimum total cost of the cement industry for 3 planning periods is 14.56 billion Tk. The model used distribution centers as collection centers. This concept reduces the setup cost and operational cost of collection centers. Result indicates that, two distribution centers have been used. The optimized network flows between facilities are shown in Table 3 to Table 6. The result shows that, for period 1, several transportation routes have been minimized by the B&B algorithm as these routes convey larger costs. The transportation routes from Narayanganj to Cox's Bazar, the transportation routes from Pabna to Dhaka, Comilla, Rajshahi, Jessore, Barisal & Cox's Bazar, and the transportation routes from Chittagong to Dhaka, Comilla, Rajshahi, & Jessore have been reduced to minimize the total cost for forward flow (Table 3).

Table 3: Optimized Network Flows from Distribution Centers to Customer Markets (Forward Flow) for the Period 1

Distribution Centers \ Customer Market	Dhaka	Comilla	Rajshahi	Jessore	Barisal	Cox's Bazar
Narayanganj	✓	✓	✓	✓	✓	×
Pabna	×	×	×	×	×	×
Chittagong	×	×	×	×	✓	✓

For forward flow and period 2, several transportation routes have been minimized by the B&B algorithm as these routes convey larger costs. The transportation routes from Narayanganj to Cox's Bazar, the transportation routes from Pabna to Dhaka, Comilla, Rajshahi, Jessore, Barisal & Cox's Bazar, the transportation routes from Chittagong to Dhaka, Comilla, Rajshahi, & Jessore have been reduced to minimize the total cost (Table 4).

Table 4: Optimized Network Flows from Distribution Centers to Customer Markets (Forward Flow) for the Period 2

Customer Market / Distribution Centers	Dhaka	Comilla	Rajshahi	Jessore	Barisal	Cox's Bazar
Narayanganj	✓	✓	✓	✓	✓	×
Pabna	×	×	×	×	×	×
Chittagong	×	×	×	×	✓	✓

For period 3, several transportation routes have been minimized by the B&B algorithm as these routes convey larger costs. The transportation routes from Narayanganj to Cox's Bazar, the transportation routes from Pabna to Dhaka, Comilla, Rajshahi, Jessore, Barisal & Cox's Bazar, the transportation routes from Chittagong to Dhaka, Comilla, Rajshahi, & Jessore have been reduced to minimize the total cost for forward flow (Table 5).

Table 5: Optimized Network Flows from Distribution Centers to Customer Markets (Forward Flow) for Period 3

Customer Market / Distribution Centers	Dhaka	Comilla	Rajshahi	Jessore	Barisal	Cox's Bazar
Narayanganj	✓	✓	×	×	✓	×
Pabna	×	×	✓	✓	×	×
Chittagong	×	×	×	×	×	✓

From Table 6, it is seen that for the period 1, several transportation routes have been minimized by the B&B algorithm from customer markets to distribution centers in case of reverse flow.

Table 6: Optimized Network Flows from Customer Markets to Distribution Centers (Reverse Flow) for the Period 1, 2 And 3

Distribution Centers / Customer Market	Narayanganj	Pabna	Chittagong
M (Dhaka)	✓	×	×
N (Comilla)	✓	×	×
O (Rajshahi)	✓	×	×
U (Jessore)	✓	×	×
V(Barisal)	✓	×	✓
W (Cox's Bazar)	×	×	✓

The transportation routes from Dhaka to Pabna & Chittagong, the transportation routes from Comilla to Pabna & Chittagong, the transportation routes from Rajshahi to Pabna & Chittagong, the transportation routes from Jessore to Pabna &

Chittagong, have been reduced to minimize the total cost for the period 1. Also, to optimize the network flows, product will flow from Barisal to Narayanganj and Chittagong. On the other hand, from Cox's Bazar only product will flow to Pabna not to the Narayanganj and the Chittagong. In case of reverse flow it is also seen that, for the period 2 and 3 the result is same as period 1 to transfer product from customer markets to distribution centers.

4. Conclusion

In this paper, a mixed integer linear programming model has been developed for single-product multi-period closed loop supply chain network problems, including manufacturing plant, distribution center and customer market. This research represents a model that may not only provide decision support to the practitioners with a minimum cost like other traditional approaches; but it also determines the optimum location of facilities along with the optimum network flows. The proposed model has been illustrated in the case of a cement industry of Bangladesh. The result clearly shows that, the designed model has successfully optimized the location of facilities and network flows which is computationally efficient. The proposed formulation and modeling approach used in this research is also applicable to different types of industries.

This research has considered the single-product problem where demand was considered as deterministic in nature. However, an industry may produce different types of products and in a real situation, demand is non-deterministic in nature. Therefore, in future, further research is needed by considering non-deterministic parameters, especially uncertainty in demand and costs of multi-product multi-period closed loop supply chain. Furthermore, for the large size problems, metaheuristics like simulated annealing, genetic algorithm, and ant colony optimization may be used to solve the problems which may lead to interesting findings.

References

Amaro, A. C. S., and Barbosa-Póvoa, A. P. D. F. S. 2007, 'Optimal planning of closed loop supply chains: a discrete versus a continuous-time formulation', *Computer Aided Chemical Engineering*, Vol. 24, No.1, Pp. 673–678.

Billal, M. M., Islam, M. M., Alam, M. S., and Hossain, M. M. 2015, 'Optimal supply chain network opportunities: the case of Bangladesh', *World Journal of Social Science*, Vol. 5, No. 3, Pp. 197-210.

Chukwuma, N. K., and Chukwuma, E. 2015, 'Optimizing cement distribution in the Nigerian cement manufacturing industry: the case of cement distribution from selected firms to markets in Ebonayi state', *International Journal of Research in Business Management*, Vol. 3, No. 2, Pp. 35-64.

Das, K., and Chowdhury, A. H. 2012, 'Designing a reverse logistics network for optimal collection, recovery and quality-based product-mix planning', *International Journal of Production Economics*, Vol. 135, No. 1, Pp. 209–221.

Dhakry, N. S., and Bangar, A. 2013, 'Minimization of inventory & transportation cost of an industry- a supply chain optimization', *Journal of Engineering Research and Applications*, Vol. 3, No. 5, Pp. 96-101.

Ene, S., and Öztürk, N. 2014, 'Open-loop reverse supply chain network design', *Procedia-Social & Behavioral Science,* Vol.109, No.2, Pp. 1110–1115.

Fleischmann, M., Beullens, P., Bloemhof-ruwaard, J. M., and Wassenhove, L. N. V. 2001, 'The impact of product recovery on logistics network design', *Production and Operations Management,* Vol. 10, No. 5, Pp. 156–173.

Georgiadis, P., Vlachos, D., and Tagaras, G. 2006, 'The impact of product lifecycle on capacity planning of closed-loop supply chains with remanufacturing', *Production & Operations Management*, Vol. 15, No. 4, Pp. 514–527.

Gou, Q., Liang, L., Huang, Z., and Xu, C. 2008, 'A joint inventory model for an open-loop reverse supply chain', *International Journal of Production Economics*, Vol.116, No. 3, Pp .28–42.

Guide, V. D. R., Harrison, T. P., and van Wassenhove, L. N. 2013, 'The challenges of closed-loop supply chains', Interfaces, Vol. 33, No. 6, Pp. 3-6.

Hu, Z., and Sheu, J. 2013, 'Post-disaster debris reverse logistics management under psychological cost minimization', *Transportation Research Part B: Methodological*, Vol. 55, Pp. 118-141.

Jayaraman, V., Patterson, R. A., and Rolland, E. 2003, 'The design of reverse distribution networks: models and solution procedures', *European Journal Operational Research*, Vol.150, No. 2, Pp.128–149.

Lee, D., and Dong, M. 2008, 'A heuristic approach to logistics network design for end-of-lease computer products recovery', *Transportation Research Part E: Logistics and Transportation Review*, Vol.44, No.3, Pp. 455-474.

Min, H., Co, C. S., and Co, H. J. 2006, 'The spatial and temporal consolidation of returned products in a closed-loop supply chain network', *Computers & Industrial Engineering*, Vol. 51, No. 2, Pp. 309–320.

Parida, S., and Andhare, A. B. 2014, 'Cost optimization of supply chain network: a case study of TMT bar manufacturing company', *International Journal of Research and Innovation in Science and Technology*, Vol. 1, No. 1, Pp. 42-48.

Roghanian, E., and Pazhoheshfar, P. 2014, 'An optimization for reverse logistics network under stochastic environment by using genetic algorithm', *Journal of Manufacturing System*, Vol. 33, Pp. 348-356.

Saffari, H., Makui, A., Mahmoodian, V., and Pishvaee, M. S. 2015, 'Multi-objective robust optimization model for social responsible closed-loop supply chain by non-dominated sorting genetic algorithm', *Journal of Industrial and Systems Engineering*, Vol. 8, No. 3, Pp. 42-59.

Sahu, V. K., and Victor, R. 2016, 'Supply chain management to minimize total transportation cost of cement plant', *International Journal of Latest Trends in Engineering and Technology*, Vol. 6, No. 3, Pp. 103-109.

Shi, L., Fan, H., Gao, P., and Zhang, H. 2009, 'Network model and optimization of medical waste logistics by improved genetic algorithm', *Advances in Computation and Intelligence*, Vol. 5821, Pp. 40-52.

Üster, H., Easwaran, G., Akçali, E., and Çetinkaya, S. 2007, 'Benders decomposition with alternative multiple cuts for a multi-product closed-loop supply chain network design model', *Naval Research Logistics,* Vol. 54, No. 6, Pp. 890–907.